Семинар 5б. Решение практических задач. Продолжение

1. Решение задачи Коши для уравнения теплопроводности.

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(k \frac{\partial u}{\partial x} \right) + f(x,t), \quad t > 0, \quad x \in (0,1), \tag{1}$$

$$u(x,0) = u_0(x), \quad x \in [0,1]; \tag{2}$$

$$u(0,t) = \mu_0(t), \quad u(1,t) = \mu_1(t), \quad t > 0. \tag{3}$$

2. Численные схемы на основе метода конечных разностей

Явная схема при постоянном k	Неявная схема при постоянном k
$\frac{u_i^{n+1} - u_i^n}{\tau} = k \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{h^2} + f_i^n$.	$\frac{u_i^{n+1} - u_i^n}{\tau} = k \frac{u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}}{h^2} + f_i^{n+1}$.
Расчетные формулы: $u_i^0 = u_0(x_i), \quad i = 0,\ldots,N_x;$ $\forall n = 0,1,2,\ldots: \quad u_0^{n+1} = \mu_0(t_{n+1}), \quad u_{N_x}^{n+1} = \mu_1(t_{n+1}),$ $u_i^{n+1} = u_i^n + \gamma\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right) + \tau f_i^n,$ $i = 1,\ldots,N_x - 1, \quad \gamma = \frac{\tau k}{h^2}.$	Расчетные формулы: $u_i^0 = u_0(x_i), \quad i = 0,\ldots,N_x;$ $\forall n = 0,1,2,\ldots: \quad u_0^{n+1} = \mu_0(t_{n+1}), \quad u_{N_x}^{n+1} = \mu_1(t_{n+1}),$ $u_i^{n+1} - \gamma\left(u_{i+1}^{n+1} - 2u_i^{n+1} + u_{i-1}^{n+1}\right) = u_i^n + \tau f_i^{n+1},$ $i = 1,\ldots,N_x - 1, \quad \gamma = \frac{\tau k}{h^2}.$

Канонический вид уравнений в случае неявной схемы:

$-A_i y_{i-1} + C_i y_i - B_i y_{i+1} = F_i, \quad 1 \le i \le N-1,$

$C_0 y_0 - B_0 y_1 = F_0, \; C_N y_N - A_N y_{N-1} = F_N,$

$y_i \equiv u_i^{n+1}, \quad i = 0,\ldots,N, \quad N \equiv N_x;$

$$A_i = B_i = \begin{cases} 0, & i = 0, \\ \gamma, & i = 1,\ldots,N-1, \\ 0, & i = N, \end{cases} \quad C_i = \begin{cases} 1, & i = 0, \\ 1+2\gamma, & i = 1,\ldots,N-1, \\ 1, & i = N, \end{cases} \quad F_i = \begin{cases} \mu_0(t_{n+1}), & i = 0, \\ u_i^n + \tau f_i^{n+1}, & i = 1,\ldots,N-1, \\ \mu_1(t_{n+1}), & i = N. \end{cases}$$

Алгоритм прогонки:

$$\alpha_0 = \frac{B_0}{C_0}, \quad \beta_0 = \frac{F_0}{C_0}, \quad \alpha_i = \frac{B_i}{C_i - A_i\alpha_{i-1}}, \quad \beta_i = \frac{F_i + A_i\beta_{i-1}}{C_i - A_i\alpha_{i-1}}, \quad i = 1,\ldots,N;$$

$$y_N = \beta_N, \quad y_i = \alpha_i y_{i+1} + \beta_i, \quad i = N-1,\ldots,0.$$

Пример:

$$f(x,t) = \exp\left[-(x-0.5)^2 / a^2 \right] \sin t, \quad a = 0.1;$$

$$u_0(x) = 1, \quad \mu_0 = 1, \quad \mu_1 = 1.$$

Текст программы для явной схемы:

```
/*============================================================*/
/* Программа "myscheme3.c"                                    */
/*============================================================*/
#include <stdio.h>
#include <stdlib.h>
#include <string.h>
#include <unistd.h>
#include <math.h>
#include "mycom.h"
```

```
double xa, xb, xc, ck, ca;

char sname[128];
char vname[12] = "uu";

double f(double x, double t);

double u0(double x);

double mu0(double t);

double mu1(double t);

void OutFile(int Nx, double *xx, double *uu, int ntv);

int main(void)
{
  int i, Nx=50, ntv;
  double Lx, hx, tau, tv, tvmax, gam;
  double *xx, *uu, *vv;
  FILE *fp;

  if ((fp = fopen("myscheme3.res", "w"))  == NULL)
  {
    printf("File not open\n");
    return 0;
  }

  xa = 0.0;
  xb = 1.0;
  xc = 0.5;
  ck = 1.0;
  ca = 0.1;

  Lx = xb-xa;
  hx = Lx/Nx;
  gam = 0.5;
  tau = gam * hx * hx / ck;
  tvmax = 3.14;

  ntv = 0;
  tv = 0;

  xx = (double*)(malloc(sizeof(double)*(Nx+1)));
  uu = (double*)(malloc(sizeof(double)*(Nx+1)));
  vv = (double*)(malloc(sizeof(double)*(Nx+1)));

  fprintf(fp,"xa=%le\n",xa);
  fprintf(fp,"xb=%le\n",xb);
  fprintf(fp,"xc=%le\n",xc);
  fprintf(fp,"ck=%le\n",ck);
  fprintf(fp,"ca=%le\n",ca);

  fprintf(fp,"Lx=%le\n",Lx);
  fprintf(fp,"Nx=%d\n",Nx);
  fprintf(fp,"hx=%le\n",hx);
  fprintf(fp,"gam=%le\n",gam);
  fprintf(fp,"tau=%le\n",tau);
  fprintf(fp,"tvmax=%le\n",tvmax);

  fprintf(fp,"\nntv=%d tv=%le\n",ntv,tv);

  for (i=0; i<=Nx; i++){
```

```
    xx[i] = xa + hx * i;
    uu[i] = u0(xx[i]);
    fprintf(fp,"i=%d x=%le u0=%le\n", i, xx[i], uu[i]);
  }

  OutFile(Nx, xx, uu, ntv);

  do {
    ntv++;
    tv += tau;

    printf("ntv=%d tv=%le\n",ntv,tv);
    fprintf(fp,"\nntv=%d tv=%le\n",ntv,tv);

    vv[0]  = mu0(tv);
    vv[Nx] = mu1(tv);

    for (i=1; i<=Nx-1; i++) {
      double ff = f(xx[i], tv-tau);
      vv[i] = uu[i] + gam * (uu[i+1] - 2.0 * uu[i] + uu[i-1]) + ff;
    }

    for (i=0; i<=Nx; i++) uu[i] = vv[i];

    if (ntv % 100 == 0) OutFile(Nx, xx, uu, ntv);

  } while (tv<=tvmax);

  fclose(fp);
  return 0;
}

double f(double x, double t)
{
  double sx = (x - xc)/ca;
  double vx = exp(-sx*sx);
  double vt = sin(t);
  return (vx*vt);
}

double u0(double x)
{
  return 1.0;
}

double mu0(double t)
{
  return 1.0;
}

double mu1(double t)
{
  return 1.0;
}

void OutFile(int Nx, double *xx, double *uu, int ntv)
{
  int i;
  FILE *F;

  for (i=0; i<128; i++) sname[i] = 0;
  sprintf(sname,"myscheme3_%s_%06d.dat",vname,ntv);

  F = fopen(sname, "wt");
```

```
//  fprintf(F,"TITLE = \"FUNC\"\n");
//  fprintf(F,"VARIABLES = \"X\", \"F\"\n");
//  fprintf(F,"ZONE I=%d, F=POINT\n",Nx+1);
  for (i=0; i<=Nx; i++) fprintf(F,"%20.13le %20.13le\n", xx[i], uu[i]);

  fclose(F);
}

Трансляция:
> gcc -o myscheme3 -O2 myscheme3.c -lm

Выполнение:
>myscheme3

>ls -l myscheme3*
-rwxr-xr-x 1 mephi99 mephi  17104 Oct 11 11:24 myscheme3
-rw-r--r-- 1 mephi99 mephi   2520 Oct 11 11:23 myscheme3.c
-rw-r--r-- 1 mephi99 mephi   6089 Oct 11 11:24 myscheme3.dem
-rwxr-xr-x 1 mephi99 mephi    168 Oct 11 09:43 myscheme3.gpl
-rw-r--r-- 1 mephi99 mephi 260941 Oct 11 11:24 myscheme3.res
-rw-r--r-- 1 mephi99 mephi   2142 Oct 11 11:24 myscheme3_uu_000000.dat
-rw-r--r-- 1 mephi99 mephi   2142 Oct 11 11:24 myscheme3_uu_000100.dat
-rw-r--r-- 1 mephi99 mephi   2142 Oct 11 11:24 myscheme3_uu_000200.dat
...

Визуальный анализ с помощью пакета gnuplot:
>myscheme3.gpl
>gnuplot myscheme3.dem
```

Задание: написать программу для неявной схемы.