## Семинар 8. Решение нелинейных пространственно одномерных краевых задач.

### 1. Постановка задачи.

$$\frac{d}{dx}\left(k(u)\frac{du}{dx}\right)-q(u)u=-f(u),\quad a<x<b,\quad u(a)=u_a,\quad u(b)=u_b.$$

Тестовый пример:

$$k(u)=1+u^2,\quad q(u)=1/(1+u^2),\quad f(u)=q(u)u-k'(u)\left(u'\right)^2-k(u)u'',$$

$$u(x)=u_a\exp\left(\alpha\frac{x-a}{b-a}\right),\quad u_b=u_a\exp(\alpha),\quad u'=\frac{\alpha}{(b-a)}u,\quad u''=\frac{\alpha^2}{(b-a)^2}u,\quad k'(u)=2u.$$

### 2. Численный алгоритм.

Разностная схема на равномерной сетке $\omega_x=\left\{x=x_i=a+ih_x,\ i=0,...,N_x,\ h_x=\frac{b-a}{N_x}\right\}$:

$$\frac{1}{\hbar_x}\left\{k_{i+1/2}\frac{y_{i+1}-y_i}{h_x}-k_{i-1/2}\frac{y_i-y_{i-1}}{h_x}\right\}-q_iy_i=-f_i,\quad 0<i<N_x,\quad y_0=u_a,\quad y_{N_x}=u_b;$$

$$k_{i\pm1/2}=\frac{k_i+k_{i\pm1}}{2},\quad k_i=k(y_i),\quad q_i=q(y_i),\quad f_i=f(y_i),\quad \hbar_x=\begin{cases}0.5h_x, & i=0,N_x,\\ h_x, & 1<i<N_x.\end{cases}$$

Канонический вид алгебраической задачи:

$$-A_i(y)y_{i-1}+C_i(y)y_i-B_i(y)y_{i+1}=F_i(y),\quad 1\le i\le N-1,$$

$$C_0(y)y_0-B_0(y)y_1=F_0(y),\quad -A_N(y)y_{N-1}+C_N(y)y_N=F_N(y).$$

Значения коэффициентов:

$$A_i(y)=\begin{cases}0, & i=0,N,\\ 0.5(k(y_i)+k(y_{i-1})), & 1\le i\le N-1,\end{cases}\quad B_i(y)=\begin{cases}0, & i=0,N,\\ 0.5(k(y_i)+k(y_{i+1})), & 1\le i\le N-1,\end{cases}$$

$$C_i(y)=\begin{cases}1, & i=0,N,\\ 0.5k(y_{i-1})+k(y_i)+0.5k(y_{i+1})+h^2q(y_i), & 1\le i\le N-1,\end{cases}\quad F_i(y)=\begin{cases}u_a, & i=0,\\ h^2f(y_i), & 1\le i\le N-1,\\ u_b, & i=N.\end{cases}$$

### 3. Метод простой итерации.

Начальное приближение: $\overset{0}{y}_i=u_a+(u_b-u_a)\frac{x_i-a}{b-a},\quad i=0,...,N.$

Расчетные формулы метода:

$$-A_i(\overset{s}{y})\overset{s+1}{y}_{i-1}+C_i(\overset{s}{y})\overset{s+1}{y}_i-B_i(\overset{s}{y})\overset{s+1}{y}_{i+1}=F_i(\overset{s}{y}),\quad 1\le i\le N-1,$$

$$C_0(\overset{s}{y})\overset{s+1}{y}_0-B_0(\overset{s}{y})\overset{s+1}{y}_1=F_0(\overset{s}{y}),\quad C_N(\overset{s}{y})\overset{s+1}{y}_N-A_N(\overset{s}{y})\overset{s+1}{y}_{N-1}=F_N(\overset{s}{y}),\quad s=0,1,...$$

Критерии сходимости:

1) по норме невязки: $\left\|\overset{s}{r}=A\overset{s}{Y}-F\right\|_C\le\varepsilon$; 2) по норме разности итераций: $\left\|\overset{s+1}{y}-\overset{s}{y}\right\|_C\le\varepsilon$.

**4. Метод Ньютона.** Расчетные формулы метода:

$$C_i(\overset{s}{y})\overset{s+1}{y}_i-A_i(\overset{s}{y})\overset{s+1}{y}_{i-1}-B_i(\overset{s}{y})\overset{s+1}{y}_{i+1}+\theta\sum_{j=0}^{N}\left[\frac{\partial C_i}{\partial y_j}(\overset{s}{y})\overset{s}{y}_i-\frac{\partial A_i}{\partial y_j}(\overset{s}{y})\overset{s}{y}_{i-1}-\frac{\partial B_i}{\partial y_j}(\overset{s}{y})\overset{s}{y}_{i+1}-\frac{\partial F_i}{\partial y_j}(\overset{s}{y})\right]\overset{s+1}{y}_j=$$

$$=F_i(\overset{s}{y})+\theta\sum_{j=0}^{N}\left[\frac{\partial C_i}{\partial y_j}(\overset{s}{y})\overset{s}{y}_i-\frac{\partial A_i}{\partial y_j}(\overset{s}{y})\overset{s}{y}_{i-1}-\frac{\partial B_i}{\partial y_j}(\overset{s}{y})\overset{s}{y}_{i+1}-\frac{\partial F_i}{\partial y_j}(\overset{s}{y})\right]\overset{s}{y}_j,\quad 1\le i\le N-1,$$

$$C_0(\overset{s}{y})\overset{s+1}{y}_0-B_0(\overset{s}{y})\overset{s+1}{y}_1+\theta\sum_{j=0}^{N}\left[\frac{\partial C_0}{\partial y_j}(\overset{s}{y})\overset{s}{y}_0-\frac{\partial B_0}{\partial y_j}(\overset{s}{y})\overset{s}{y}_1-\frac{\partial F_0}{\partial y_j}(\overset{s}{y})\right]\overset{s+1}{y}_j=F_0(\overset{s}{y})+\theta\sum_{j=0}^{N}\left[\frac{\partial C_0}{\partial y_j}(\overset{s}{y})\overset{s}{y}_0-\frac{\partial B_0}{\partial y_j}(\overset{s}{y})\overset{s}{y}_1-\frac{\partial F_0}{\partial y_j}(\overset{s}{y})\right]\overset{s}{y}_j,$$

$$C_N(\overset{s}{y})\overset{s+1}{y}_N-A_N(\overset{s}{y})\overset{s+1}{y}_{N-1}+\theta\sum_{j=0}^{N}\left[\frac{\partial C_N}{\partial y_j}(\overset{s}{y})\overset{s}{y}_N-\frac{\partial A_N}{\partial y_j}(\overset{s}{y})\overset{s}{y}_{N-1}-\frac{\partial F_N}{\partial y_j}(\overset{s}{y})\right]\overset{s+1}{y}_j=$$

$$=F_N(\overset{s}{y})+\theta\sum_{j=0}^{N}\left[\frac{\partial C_N}{\partial y_j}(\overset{s}{y})\overset{s}{y}_N-\frac{\partial A_N}{\partial y_j}(\overset{s}{y})\overset{s}{y}_{N-1}-\frac{\partial F_N}{\partial y_j}(\overset{s}{y})\right]\overset{s}{y}_j,\quad s=0,1,...,\quad 0\le\theta\le1.$$

В случае ***локальной нелинейности*** (когда коэффициенты зависят только от самой функции $u$) получаем во внутренних узлах:

$$\tilde{C}_i(\overset{s}{y})\,\overset{s+1}{y}_i-\tilde{A}_i(\overset{s}{y})\,\overset{s+1}{y}_{i-1}-\tilde{B}_i(\overset{s}{y})\,\overset{s+1}{y}_{i+1}=\tilde{F}_i(\overset{s}{y}),\quad 1\le i\le N-1,$$

$$\tilde{A}_i(\overset{s}{y})=A_i(\overset{s}{y})-\theta\left[\frac{\partial C_i}{\partial y_{i-1}}(\overset{s}{y})\,\overset{s}{y}_i-\frac{\partial A_i}{\partial y_{i-1}}(\overset{s}{y})\,\overset{s}{y}_{i-1}\right],\quad \tilde{B}_i(\overset{s}{y})=B_i(\overset{s}{y})-\theta\left[\frac{\partial C_i}{\partial y_{i+1}}(\overset{s}{y})\,\overset{s}{y}_i-\frac{\partial B_i}{\partial y_{i+1}}(\overset{s}{y})\,\overset{s}{y}_{i+1}\right],$$

$$\tilde{C}_i(\overset{s}{y})=C_i(\overset{s}{y})+\theta\left[\frac{\partial C_i}{\partial y_i}(\overset{s}{y})\,\overset{s}{y}_i-\frac{\partial A_i}{\partial y_i}(\overset{s}{y})\,\overset{s}{y}_{i-1}-\frac{\partial B_i}{\partial y_i}(\overset{s}{y})\,\overset{s}{y}_{i+1}-\frac{\partial F_i}{\partial y_i}(\overset{s}{y})\right],$$

$$\tilde{F}_i(\overset{s}{y})=F_i(\overset{s}{y})+\theta\left[\frac{\partial C_i}{\partial y_i}(\overset{s}{y})\,\overset{s}{y}_i-\frac{\partial A_i}{\partial y_i}(\overset{s}{y})\,\overset{s}{y}_{i-1}-\frac{\partial B_i}{\partial y_i}(\overset{s}{y})\,\overset{s}{y}_{i+1}-\frac{\partial F_i}{\partial y_i}(\overset{s}{y})\right]\overset{s}{y}_i+$$

$$+\theta\left[\frac{\partial C_i}{\partial y_{i-1}}(\overset{s}{y})\,\overset{s}{y}_i-\frac{\partial A_i}{\partial y_{i-1}}(\overset{s}{y})\,\overset{s}{y}_{i-1}\right]\overset{s}{y}_{i-1}+\theta\left[\frac{\partial C_i}{\partial y_{i+1}}(\overset{s}{y})\,\overset{s}{y}_i-\frac{\partial B_i}{\partial y_{i+1}}(\overset{s}{y})\,\overset{s}{y}_{i+1}\right]\overset{s}{y}_{i+1};$$

На левой границе:

$$\tilde{C}_0(\overset{s}{y})\,\overset{s+1}{y}_0-\tilde{B}_0(\overset{s}{y})\,\overset{s+1}{y}_1=\tilde{F}_0(\overset{s}{y}),$$

$$\tilde{B}_0(\overset{s}{y})=B_0(\overset{s}{y})-\theta\left[\frac{\partial C_0}{\partial y_1}(\overset{s}{y})\,\overset{s}{y}_0-\frac{\partial B_0}{\partial y_1}(\overset{s}{y})\,\overset{s}{y}_1\right],\quad \tilde{C}_0(\overset{s}{y})=C_0(\overset{s}{y})+\theta\left[\frac{\partial C_0}{\partial y_0}(\overset{s}{y})\,\overset{s}{y}_0-\frac{\partial B_0}{\partial y_0}(\overset{s}{y})\,\overset{s}{y}_1-\frac{\partial F_0}{\partial y_0}(\overset{s}{y})\right],$$

$$\tilde{F}_0(\overset{s}{y})=F_0(\overset{s}{y})+\theta\left[\frac{\partial C_0}{\partial y_0}(\overset{s}{y})\,\overset{s}{y}_0-\frac{\partial B_0}{\partial y_0}(\overset{s}{y})\,\overset{s}{y}_1-\frac{\partial F_0}{\partial y_0}(\overset{s}{y})\right]\overset{s}{y}_0+\theta\left[\frac{\partial C_0}{\partial y_1}(\overset{s}{y})\,\overset{s}{y}_0-\frac{\partial B_0}{\partial y_1}(\overset{s}{y})\,\overset{s}{y}_1\right]\overset{s}{y}_1;$$

На правой границе:

$$\tilde{C}_N(\overset{s}{y})\,\overset{s+1}{y}_N-\tilde{A}_N(\overset{s}{y})\,\overset{s+1}{y}_{N-1}=\tilde{F}_N(\overset{s}{y}),$$

$$\tilde{A}_N(\overset{s}{y})=A_N(\overset{s}{y})-\theta\left[\frac{\partial C_N}{\partial y_{N-1}}(\overset{s}{y})\,\overset{s}{y}_N-\frac{\partial A_N}{\partial y_{N-1}}(\overset{s}{y})\,\overset{s}{y}_{N-1}\right],\quad \tilde{C}_N(\overset{s}{y})=C_N(\overset{s}{y})+\theta\left[\frac{\partial C_N}{\partial y_N}(\overset{s}{y})\,\overset{s}{y}_N-\frac{\partial A_N}{\partial y_N}(\overset{s}{y})\,\overset{s}{y}_{N-1}-\frac{\partial F_N}{\partial y_N}(\overset{s}{y})\right],$$

$$\tilde{F}_N(\overset{s}{y})=F_N(\overset{s}{y})+\theta\left[\frac{\partial C_N}{\partial y_N}(\overset{s}{y})\,\overset{s}{y}_N-\frac{\partial A_N}{\partial y_N}(\overset{s}{y})\,\overset{s}{y}_{N-1}-\frac{\partial F_N}{\partial y_N}(\overset{s}{y})\right]\overset{s}{y}_N+\theta\left[\frac{\partial C_N}{\partial y_{N-1}}(\overset{s}{y})\,\overset{s}{y}_N-\frac{\partial A_N}{\partial y_{N-1}}(\overset{s}{y})\,\overset{s}{y}_{N-1}\right]\overset{s}{y}_{N-1}.$$

Ненулевые производные:

$$\frac{\partial A_i}{\partial y_i}(\overset{s}{y})=0.5\frac{\partial k}{\partial u}(\overset{s}{y}_i),\quad \frac{\partial A_i}{\partial y_{i-1}}(\overset{s}{y})=0.5\frac{\partial k}{\partial u}(\overset{s}{y}_{i-1}),\quad i=1,...,N;$$

$$\frac{\partial B_i}{\partial y_i}(\overset{s}{y})=0.5\frac{\partial k}{\partial u}(\overset{s}{y}_i),\quad \frac{\partial B_i}{\partial y_{i+1}}(\overset{s}{y})=0.5\frac{\partial k}{\partial u}(\overset{s}{y}_{i+1}),\quad i=0,...,N-1;$$

$$\frac{\partial C_i}{\partial y_i}(\overset{s}{y})=\frac{\partial k}{\partial u}(\overset{s}{y}_i)+h^2\frac{\partial q}{\partial y}(\overset{s}{y}_i),\quad \frac{\partial C_i}{\partial y_{i-1}}(\overset{s}{y})=0.5\frac{\partial k}{\partial u}(\overset{s}{y}_{i-1}),\quad \frac{\partial C_i}{\partial y_{i+1}}(\overset{s}{y})=0.5\frac{\partial k}{\partial u}(\overset{s}{y}_{i+1}),\quad i=1,...,N-1;$$

$$\frac{\partial C_0}{\partial y_0}(\overset{s}{y})=\frac{\partial k}{\partial u}(\overset{s}{y}_0)+h^2\frac{\partial q}{\partial y}(\overset{s}{y}_0),\quad \frac{\partial C_0}{\partial y_1}(\overset{s}{y})=0.5\frac{\partial k}{\partial u}(\overset{s}{y}_1);$$

$$\frac{\partial C_N}{\partial y_N}(\overset{s}{y})=\frac{\partial k}{\partial u}(\overset{s}{y}_N)+h^2\frac{\partial q}{\partial y}(\overset{s}{y}_N),\quad \frac{\partial C_N}{\partial y_{N-1}}(\overset{s}{y})=0.5\frac{\partial k}{\partial u}(\overset{s}{y}_{N-1});$$

$$\frac{\partial F_i}{\partial y_i}(\overset{s}{y})=h^2\frac{\partial f}{\partial y}(\overset{s}{y}_i),\quad i=0,...,N.$$

Если $k(u)\equiv 1$, то

$$\frac{\partial A_i}{\partial y_j}(\overset{s}{y})=0,\quad \frac{\partial B_i}{\partial y_j}(\overset{s}{y})=0,\quad \frac{\partial C_i}{\partial y_i}(\overset{s}{y})=h^2\frac{\partial q}{\partial y}(\overset{s}{y}_i),\quad \frac{\partial F_i}{\partial y_i}(\overset{s}{y})=h^2\frac{\partial f}{\partial y}(\overset{s}{y}_i),$$

$$\tilde{C}_i(\overset{s}{y})=C_i(\overset{s}{y})+\theta\left(\frac{\partial C_i}{\partial y_i}(\overset{s}{y})\,\overset{s}{y}_i-\frac{\partial F_i}{\partial y_i}(\overset{s}{y})\right),\quad \tilde{F}_i(\overset{s}{y})=F_i(\overset{s}{y})+\theta\left(\frac{\partial C_i}{\partial y_i}(\overset{s}{y})\,\overset{s}{y}_i-\frac{\partial F_i}{\partial y_i}(\overset{s}{y})\right)\overset{s}{y}_i.$$

## 5. Реализация примеров.

**Пример 1. Решение задачи методом простой итерации (ex12a.c):**

```
#include <stdio.h>  #include <stdlib.h>  #include <string.h>  #include <math.h>
#include "mycom.h"  #include "mynet.h"  #include "myprog.h"
static int np, mp, nl, ier, lp;  static int np1, np2, mp1, mp2;
```

```
static char pname[MPI_MAX_PROCESSOR_NAME];  static char sname[10] = "ex12a.p00";
static MPI_Status status;  static union_t buf;  static double tick, t1, t2, t3;
static FILE *Fi = NULL, *Fo = NULL;
static int nx, it, itm;
static double xa, xb, ua, ub, alf, eps, pa, pa2, uc, rka;
double k(double u); double k(double u) { return 1.0 + u*u; }
double k1(double u); double k1(double u) { return 2.0*u; }
double q(double u); double q(double u) { return 1.0 / (1.0 + u*u); }
double u(double x); double u(double x) { return ua*exp(pa*(x-xa)); }
double u1(double u); double u1(double u) { return pa*u; }
double u2(double u); double u2(double u) { return pa2*u; }
double f(double u); double f(double u) { return q(u)*u - k1(u)*u1(u)*u1(u) - k(u)*u2(u);}
int main(int argc, char *argv[])
{
  int i, j, i1, i2, nc, ncm, ncp, ncx;
  double hx, hx2, s0, s1, s2, sm, sp, y0m, y0p;
  double *xx, *aa, *bb, *cc, *dd, *ee, *ff, *al, *y0, *y1, *y2, *y3, *y4;
  MyNetInit(&argc,&argv,&np,&mp,&nl,pname,&tick);
  fprintf(stderr,"Netsize: %d, process: %d, system: %s, tick=%12le\n",np,mp,pname,tick);
  sleep(1);
  sprintf(sname+7,"%02d",mp);
  ier = fopen_m(&Fo,sname,"wt");
  if (ier!=0) mpierr("Protocol file not opened",1);
  if (mp==0) {
    ier = fopen_m(&Fi,"ex12a.d","rt"); if (ier!=0) mpierr("Data file not opened",2);
    fscanf(Fi,"xa=%le\n",&xa); fscanf(Fi,"xb=%le\n",&xb);
    fscanf(Fi,"ua=%le\n",&ua); fscanf(Fi,"ub=%le\n",&ub);
    fscanf(Fi,"alf=%le\n",&alf); fscanf(Fi,"eps=%le\n",&eps);
    fscanf(Fi,"itm=%d\n",&itm); fscanf(Fi,"nx=%d\n",&nx);
    fscanf(Fi,"lp=%d\n",&lp); fclose_m(&Fi);
    if (argc>1) sscanf(argv[1],"%d",&nx);
  }
  if (np>1) {
    if (mp==0) {
      buf.ddata[0] = xa; buf.ddata[1] = xb; buf.ddata[2] = ua; buf.ddata[3] = ub;
      buf.ddata[4] = alf; buf.ddata[5] = eps;
      buf.idata[12] = itm; buf.idata[13] = nx; buf.idata[14] = lp;
    }
    MPI_Bcast(buf.ddata,8,MPI_DOUBLE,0,MPI_COMM_WORLD);
    if (mp>0) {
      xa = buf.ddata[0]; xb = buf.ddata[1]; ua = buf.ddata[2]; ub = buf.ddata[3];
      alf = buf.ddata[4]; eps = buf.ddata[5];
      itm = buf.idata[12]; nx = buf.idata[13]; lp = buf.idata[14];
    }
  }
  ub = ua * exp(alf);
  fprintf(Fo,"Netsize: %d, process: %d, system: %s, tick=%12le\n", np,mp,pname,tick);
  fprintf(Fo,"xa=%le xb=%le ua=%le ub=%le alf=%le eps=%le itm=%d nx=%d lp=%d\n",
    xa,xb,ua,ub,alf,eps,itm,nx,lp);
  t1 = MPI_Wtime();
  pa = alf / (xb-xa); pa2 = pa*pa; uc = (ub-ua) / (xb-xa);
  hx = (xb-xa)/nx; hx2 = hx * hx;
  MyRange(np,mp,0,nx,&i1,&i2,&nc);
  ncm = nc-1; ncp = 2*(np-1); ncx = imax(nc,ncp);
  fprintf(Fo,"i1=%d i2=%d nc=%d\n",i1,i2,nc);
  xx = (double*)(malloc(sizeof(double)*nc)); y0 = (double*)(malloc(sizeof(double)*nc));
  y1 = (double*)(malloc(sizeof(double)*nc)); aa = (double*)(malloc(sizeof(double)*ncx));
  bb = (double*)(malloc(sizeof(double)*ncx)); cc = (double*)(malloc(sizeof(double)*ncx));
  ff = (double*)(malloc(sizeof(double)*ncx)); al = (double*)(malloc(sizeof(double)*ncx));
  if (np>1) {
    y2 = (double*)(malloc(sizeof(double)*nc)); y3 = (double*)(malloc(sizeof(double)*nc));
    y4 = (double*)(malloc(sizeof(double)*ncp));
    dd = (double*)(malloc(sizeof(double)*4*ncp));
    ee = (double*)(malloc(sizeof(double)*4*ncp));
  }
  for (i=0; i<nc; i++) xx[i] = xa + hx * (i1 + i); // grid
  it = 0;
  for (i=0; i<nc; i++) y1[i] = ua + uc * (xx[i]-xa); // start solution
```

```
// Iterations:
  do {
    for (i=0; i<nc; i++) y0[i] = y1[i];
    if (np>1)
      BndExch1D(np,mp,1,1,1,1,&(y0[0]),&y0m,&(y0[ncm]),&y0p);
    else { y0m = 0.0; y0p = 0.0; }
    if (mp==0) { aa[0] = 0.0; bb[0] = 0.0; cc[0] = 1.0; ff[0] = ua; }
    else {
      s0 = k(y0[0]); s1 = k(y0m); s2 = k(y0[1]);
      aa[0] = 0.5 * (s0 + s1); bb[0] = 0.5 * (s0 + s2);
      cc[0] = hx2 * q(y0[0]) + aa[0] + bb[0]; ff[0] = hx2 * f(y0[0]);
    }
    for (i=1; i<ncm; i++) {
      s0 = k(y0[i]); s1 = k(y0[i-1]); s2 = k(y0[i+1]);
      aa[i] = 0.5 * (s0 + s1); bb[i] = 0.5 * (s0 + s2);
      cc[i] = hx2 * q(y0[i]) + aa[i] + bb[i]; ff[i] = hx2 * f(y0[i]);
    }
    if (mp==np-1) { aa[ncm] = 0.0; bb[ncm] = 0.0; cc[ncm] = 1.0; ff[ncm] = ub; }
    else {
      s0 = k(y0[ncm]); s1 = k(y0[ncm-1]); s2 = k(y0p);
      aa[ncm] = 0.5 * (s0 + s1); bb[ncm] = 0.5 * (s0 + s2);
      cc[ncm] = hx2 * q(y0[ncm]) + aa[ncm] + bb[ncm]; ff[ncm] = hx2 * f(y0[ncm]);
    }
    rka = 0.0;
    for (i=0; i<nc; i++) {
      s0 = y0[i];
      if (i==  0) { if (mp>   0) sm = y0m; else sm = 0.0; } else sm = y0[i-1];
      if (i==ncm) { if (mp<np-1) sp = y0p; else sp = 0.0; } else sp = y0[i+1];
      s1 = ff[i] + aa[i] * sm + bb[i] * sp - cc[i] * s0;
      rka = dmax(rka,dabs(s1));
    }
    if (np>1) { s0 = rka; MPI_Allreduce(&s0,&rka,1,MPI_DOUBLE,MPI_MAX,MPI_COMM_WORLD); }
    if ((lp>0) && (mp==0)) fprintf(stderr,"it=%d rka=%le\n",it,rka);
    if (rka<=eps) break;
    it++;
    if (np<2) ier = prog_right(nc,aa,bb,cc,ff,al,y1);
    else      ier = prog_rightp(np,mp,nc,aa,bb,cc,ff,al,y1,y2,y3,y4,dd,ee);
    if (ier!=0) mpierr("Bad solution",1);
  } while (it<=itm);
  t1 = MPI_Wtime() - t1;
  s0 = 0.0;
  for (i=0; i<nc; i++) {
    s1 = u(xx[i]); s2 = dabs(s1-y1[i]); s0 = dmax(s0,s2);
    if (lp>0) fprintf(Fo,"i=%8d x=%12le y=%12le u=%12le d=%12le\n",i,xx[i],y1[i],s1,s2);
  }
  if (np>1) { s1 = s0; MPI_Allreduce(&s1,&s0,1,MPI_DOUBLE,MPI_MAX,MPI_COMM_WORLD); }
  if (mp==0) fprintf(stderr,"nx=%d it=%d time=%le dmax=%le\n",nx,it,t1,s0);
  fprintf(Fo,"nx=%d it=%d time=%le dmax=%le\n",nx,it,t1,s0);
  ier = fclose_m(&Fo);
  MPI_Finalize();
  return 0;
}
```

Трансляция:

```
>mpicc -o ex12a.px -O2 ex12a.c mycom.c mynet.c myprog.c -lm
```

Результаты выполнения:

```
np=1 nx=10      it=30 time=5.400000e-05 dmax=2.830711e-03
np=1 nx=100     it=24 time=4.170000e-04 dmax=2.869907e-05
np=1 nx=1000    it=20 time=3.491000e-03 dmax=2.871838e-07
np=1 nx=10000   it=16 time=2.766900e-02 dmax=1.581609e-08
np=1 nx=100000  it=12 time=2.604180e-01 dmax=9.598133e-07
np=1 nx=1000000 it=7  time=1.512643e+00 dmax=2.028339e-04
np=2 nx=1000000 it=7  time=1.063321e+00 dmax=2.028401e-04
np=3 nx=1000000 it=7  time=9.272730e-01 dmax=2.028154e-04
np=4 nx=1000000 it=7  time=6.390440e-01 dmax=2.028409e-04
np=5 nx=1000000 it=7  time=6.064660e-01 dmax=2.028184e-04
np=6 nx=1000000 it=7  time=5.269030e-01 dmax=2.028220e-04
```