Iterative Method for Matching Solutions to the Heat Balance Equations for Different Parts of a Thermoelectric Cooler

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1. Introduction

The temperature distribution in a thermoelectric cooling module is considered. The module is an array of thermocouples made of n- and p-type semiconductors and connected electrically in series. An iterative algorithm based on a domain decomposition is proposed. Temperature distributions are computed for subdomains characterized by different physical properties and nonlinear temperature dependencies of the coefficients contained in the heat balance equation. The resulting analytical and numerical solutions are matched on the subdomain boundaries.

2. Statement of the problem

In this report we propose a method for solving the heat balance equations describing the temperature distribution in a thermoelectric cooler.

A thermoelectric cooling module is a multi-element assembly of components characterized by different coefficients of thermal expansion, including thermocouples made of n- and p-type semiconductor branches (connected electrically in series and thermally in parallel) and solder layers (Fig.1). When a direct current is passed through the thermocouples, heat is absorbed at certain thermocouple junctions and evolved at other junctions (this phenomenon is known as the Peltier effect) [1].

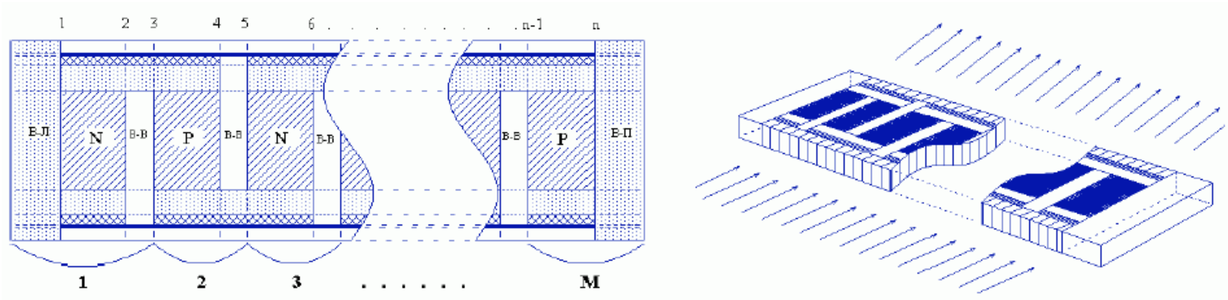

Fig. 1

In [2-4], temperature distributions were obtained for different components of thermoelectric cooling modules by calculating temperature at each grid point. Since a thermoelectric cooling module consists of many identical units, substantial computing and storage resources are required to perform the calculations for the module as a whole, and it is reasonable to explore the possibility of performing parallel computations for these units on a multiprocessor computer.

Accordingly, the entire domain to be considered was decomposed into nonoverlapping subdomains. In this approach, the subdomain boundaries were defined, and the method was developed for matching the corresponding solutions. Similar approached have been developed in numerous studies of boundary value problems for elliptic equations. The analyses presented in [5,6] should be mentioned here as the guidelines for our study.

In this report, we present an iterative method based on decomposition into subdomains and matching of solutions on their boundaries. The method is designed to solve the specific problem outlined above. Our analysis is focused on the development of a computational tech-

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nique for simulating the heat transfer processes that take place in a thermoelectric module through parallized computations for individual units on a multiprocessor computer.

3. System of equations for heat transfer in the module element and numerical solutions method

A time-independent temperature distribution in a nonuniformly heated thermoelectric material is governed by the equation

$$\operatorname{div}\left(\kappa\nabla T\right)+\rho j^{2}-Tj\nabla\alpha=0\,, \tag{1}$$

where κ is thermal conductivity, ρ is resistivity, j is current density, and α is the thermoelectric coefficient.

Eq.1 can be rewritten in equivalent form as

$$\nabla\kappa\nabla T+\rho j^{2}-\tau j\nabla T=0\,, \tag{2}$$

where $\tau=T\,\partial\alpha/\partial T$ is the Thomson coefficient.

In a two-dimensional Cartesian coordinate system denoting by $T^{(n)}$ the temperature calculated as a result of the n-th iteration step, we can rewrite Eq. (2) as follows:

$$\left(\alpha^{(n)}-\frac{\partial}{\partial x}\kappa\frac{\partial}{\partial x}\right)\left(\alpha^{(n)}-\frac{\partial}{\partial y}\kappa\frac{\partial}{\partial y}\right)\left(T^{(n+1)}-T^{(n)}\right)=\alpha^{(n)}\omega\left[L\left(T^{(n)}\right)-f\left(T^{(n)}\right)\right] \tag{3}$$

Defining an auxiliary quantity T^{*} as the intermediate value of temperature corresponding to an intermediate step and applying the approximate factorization method, we represented the transition from the n-th to (n+1)-th iteration step as a sequence of two steps:

$$\left(\alpha^{(n)}-\frac{\partial}{\partial x}\kappa\frac{\partial}{\partial x}\right)T^{*}=\alpha^{(n)}\omega\left[L\left(T^{(n)}\right)-f\left(T^{(n)}\right)\right] \tag{4a}$$

$$\left(\alpha^{(n)}-\frac{\partial}{\partial y}\kappa\frac{\partial}{\partial y}\right)\left(T^{(n+1)}-T^{(n)}\right)=T^{*} \tag{4b}$$

Both equations were solved by using the scalar flux-based tridiagonal algorithm.

In a set of subdomains the temperature fields are determined analytically. The analytical solution for temperature in each of such subdomains has the form

$$\begin{aligned}T\left(\tilde{x},\tilde{y}\right)=T_{1}+\frac{T_{2}-T_{1}}{p}\tilde{x}+\frac{T_{3}-T_{1}}{p}\tilde{y}+\frac{T_{4}-T_{2}-T_{3}+T_{1}}{pq}\tilde{x}\tilde{y}+\\+\sqrt{\frac{2}{p}}\sum_{k=1}^{\infty}\frac{\sin\left(\pi k\tilde{x}/p\right)}{\operatorname{sh}\left(\pi kq/p\right)}\left\{a_{k}\operatorname{sh}\frac{\pi k\left(q-\tilde{y}\right)}{p}+b_{k}\operatorname{sh}\frac{\pi k\tilde{y}}{p}\right\}+\\+\sqrt{\frac{2}{p}}\sum_{k=1}^{\infty}\frac{\sin\left(\pi k\tilde{y}/q\right)}{\operatorname{sh}\left(\pi kp/q\right)}\left\{c_{k}\operatorname{sh}\frac{\pi k\left(p-\tilde{x}\right)}{q}+d_{k}\operatorname{sh}\frac{\pi k\tilde{x}}{q}\right\}-\frac{Q}{\kappa}\frac{\tilde{y}^{2}}{2},\end{aligned} \tag{5}$$

This technique was employed to solve the problem on a multiprocessor computer “Supercomputer MVS 1000M”. The solutions computed by means of different processors should be matched along the vertical boundaries that separate the domains of analytical solutions from the semiconductor domains.

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