An Example to the Numerical Processing of Nonisothermal Flow of non-Newtonian Fluids

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1 Introduction

In most polymer processing applications and in lubrication systems the changes of temperature are significant and cannot be ignored. The description of nonisothermal flows not only requires the simultaneous solution of three equations of change (continuity, motion, and energy), but in addition the temperature dependence of all physical properties, especially that of the viscosity, must in general be taken into account; moreover, for polymeric liquids the shear-rate dependence of the viscosity cannot be neglected (Bird 1987). As an example, we shall treat the flow of a non-Newtonian fluid round a hot sphere falling along the centreline of a cylindrical tube, with using of parallel computers.

The well – known equations for the fluid-flow and the heat – transfer are as follows :

Continuity $\nabla . V = 0$ (1)

Motion $$\rho \frac{DV}{Dt} = \rho g - \nabla p + \nabla \eta \dot{\gamma} \quad (2)$$

Energy $$\rho C_p \frac{DT}{Dt} = \nabla . k \nabla T + \frac{1}{2} \eta \left(\dot{\gamma} : \dot{\gamma} \right) \quad (3)$$

Due to the temperature – dependence of the viscosity and constitutive equation coefficients, the equation of motion and energy are strongly coupled. In general, then, the solution of the set of equations of change cannot be obtained analytically. This problem was resolved for the isothermal case using Finite Element Method (Celasun et al. 2002).

2 Isothermal case

In the isothermal case the schematic diagram of a sphere of radius a falling through a non – Newtonian fluid in a cylinder of radius R is given in Fig.1. The global cylindrical coordinate system is (r, θ, z). The local area coordinates are L_1, L_2, L_3. The notations are the usual ones. Let's see promptly the formulation in that case.

2.1 The non-Newtonian fluid chosen (CEF Model)

The constitutive equation of the CEF fluid is

$$\tau = -pI + \eta A_1 + \left(v_1 + v_2 \right) A_1^2 - \frac{1}{2} v_1 A_2 \quad (4)$$

The Rivlin-Ericksen tensors involved in the CEF equation are

$$A_1 = 2d = \nabla V + \nabla V^T$$

$$A_1^2 = 4d^2$$

$$A_2 = \nabla a + \nabla a^T + 2 \nabla V \bullet \nabla V^T$$

The shear strain-rate in term of the second invariant II_d is

$$\dot{\gamma} = 2\sqrt{II_d} = \sqrt{\frac{1}{2} tr A_1^2} = \sqrt{\frac{1}{2} tr\, A_2}$$

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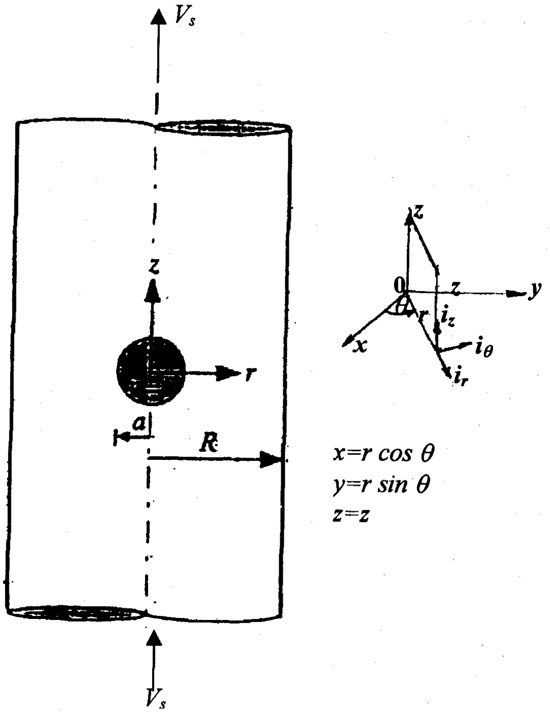

Fig. 1 Schematic diagram of a sphere falling through a fluid in a cylinder.

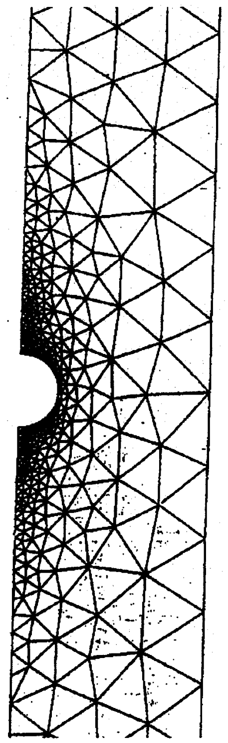

Fig. 2 Mesh pattern around sphere, *a/R*= 0.2

3 Material functions in steady-state shear flows

Viscosity : $\tau_{yx} = \eta(\dot{\gamma})\dot{\gamma}_{yx}$

Normal stress coefficients :

$$\tau_{xx} - \tau_{yy} = \upsilon_1(\dot{\gamma})\dot{\gamma}^2$$

$$\tau_{yy} - \tau_{zz} = \upsilon_2(\dot{\gamma})\dot{\gamma}^2$$

Carreau formula for the viscosity coefficient :

$$\eta = \eta_0\left(1 + 32.32\, tr\, A_1^2\right)^{-0.318}$$

$$\frac{v_1}{\eta} = 10^{\left[-0.169(\log_{10}\dot{\gamma})^2 - 0.76\log_{10}\dot{\gamma} - 0.821\right]}$$

$$v_2 = -0.15 v_1 \quad \text{and thus} \quad v_1 + v_2 = 0.85 v_1$$

4 Explicit expressions of the dimensionless governing equations

Continuity equation

$$\frac{\partial v_r}{\partial r} + \frac{\partial v_z}{\partial z} + \frac{v_r}{r} = 0 \tag{5}$$

Projection of the motion equation on the *r* axis

$$-\frac{\partial p}{\partial r} + \frac{\partial \eta}{\partial r}(A_1)_{rr} + \frac{\partial \eta}{\partial z}(A_1)_{zr} + \eta\left[\frac{\partial (A_1)_{rr}}{\partial r} + \frac{\partial (A_1)_{zr}}{\partial z} + \frac{(A_1)_{rr} - (A_1)_{\theta\theta}}{r}\right] +$$
$$+ 0.85K\left\{\frac{\partial \upsilon_1}{\partial r}(A_1^2)_{rr} + \frac{\partial \upsilon_1}{\partial z}(A_1^2)_{zr} + \upsilon_1\left[\frac{\partial (A_1^2)_{rr}}{\partial r} + \frac{\partial (A_1^2)_{zr}}{\partial z} + \frac{(A_1^2)_{rr} - (A_1^2)_{\theta\theta}}{r}\right]\right\} - \tag{6}$$
$$-\frac{1}{2}K\left\{\frac{\partial \upsilon_1}{\partial r}(A_2)_{rr} + \frac{\partial \upsilon_1}{\partial z}(A_2)_{zr} + \upsilon_1\left[\frac{\partial (A_2)_{rr}}{\partial r} + \frac{\partial (A_2)_{zr}}{\partial z} + \frac{(A_2)_{rr} - (A_2)_{\theta\theta}}{r}\right]\right\} = 0$$

Projection of the motion equation on the z axis

$$-\frac{\partial p}{\partial z}+\frac{\partial \eta}{\partial r}(A_1)_{rz}+\frac{\partial \eta}{\partial z}(A_1)_{zz}+\eta\left[\frac{\partial (A_1)_{rz}}{\partial r}+\frac{\partial (A_1)_{zz}}{\partial z}+\frac{(A_1)_{rz}}{r}\right]+$$
$$+0.85K\left\{\frac{\partial \upsilon_1}{\partial r}(A_1^2)_{rz}+\frac{\partial \upsilon_1}{\partial z}(A_1^2)_{zz}+\upsilon_1\left[\frac{\partial (A_1^2)_{rz}}{\partial r}+\frac{\partial (A_1^2)_{zz}}{\partial z}+\frac{(A_1^2)_{rz}}{r}\right]\right\}-$$
$$-\frac{1}{2}K\left\{\frac{\partial \upsilon_1}{\partial r}(A_2)_{rz}+\frac{\partial \upsilon_1}{\partial z}(A_2)_{zz}+\upsilon_1\left[\frac{\partial (A_2)_{rz}}{\partial r}+\frac{\partial (A_2)_{zz}}{\partial z}+\frac{(A_2)_{rz}}{r}\right]\right\}=0 \quad (7)$$

5 Nonisothermal case

Admitting combined hydrodynamic and thermal entry length solution in the asymptotic zone and neglecting energy dissipation and axial conduction $k\frac{\partial^2 T}{\partial z^2}$ term, the energy equation (3) becomes:

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}=\frac{v_z(r)}{\chi}\frac{\partial T}{\partial z} \quad (8)$$

The thermal diffusivity $\chi = k/\rho C_p$, the wall temperature T_w, and the hot sphere temperature T_s are supposed constant. The entry temperature is T_∞ and the uniform entry velocity is V_s. Dimensionless temperature is $\theta=(T-T_w)/(T_s-T_w)$.

The dimensionless boundary conditions are $\theta=1$ at $\sqrt{r^2+z^2}=1$,
$\theta=0$ at R/a and $\theta=(T_\infty-T_w)/(T_s-T_w)$ at $z=\infty$.

The dimensionless form of the eq. (8), with the Péclet number Rv_z/χ is

$$\frac{Pé}{R}\frac{\partial \theta}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) \quad (9)$$

6 Temperature effects

As temperature is increased, the zero-shear-rate viscosity η_o decreases. Considering the "master curve" and the "shift factor a_T" concepts, the viscosity measured at a temperature T and shear rate $\dot{\gamma}$ is equivalent, after correction for the temperature dependence of η_o, to viscosity measured at the reference temperature T_o and shear rate $a_T\dot{\gamma}$.

According to "Arrhenius dependence" :

$$a_T=\exp\left[\frac{E}{R}\left(\frac{1}{T}-\frac{1}{T_o}\right)\right] \quad (10)$$

The flow activation energy ratio E/R has typical values around 4.5 x 10^3 K. We chose as reference temperature T_o the usual inlet temperature T_∞. The formulas applied are:

$$\eta(\dot{\gamma},T)=\eta(\dot{\gamma},T_o)a_T T/T_o \quad \text{and} \quad \upsilon_1(\dot{\gamma},T)=\upsilon_1(\dot{\gamma},T_o)a_T^2 T/T_o \quad (11)$$

7 Conclusions

We chose a linear triangle for the temperatures, like for pressures in the Finite Element process whereas we take a quadratic one for the velocities (Fig. 2). Thus, together with the continuity and motion equations (5), (6), (7) and now the energy equation (9), we have at each triangle overall 18 equations and 18 variables as v_r , v_z, p and T (we don't use eqs (5) and (9) at midside nodes). We apply the same procedure as for the isothermal case using always the area coordinates and the Gauss quadrature. We have numerous simultaneous and highly non linear equations, which moreover are overdetermined due to the existence of the boundary conditions. Hence, we were compelled to resort to the optimisation techniques to resolve the equation system. Fig.3 displays the great distortion of the flow field that is possible due to thermal effects (Morris 1982).

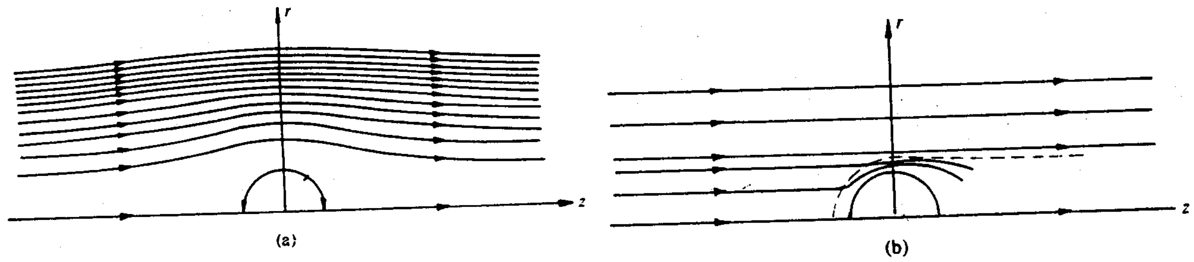

Fig. 3 Effect on streamlines of viscosity –temperature variation in Newtonian, creeping flow. (a) Isothermal solution. (b) Hot sphere, heat-sensitive viscosity.

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