Background of Computations for Mathematical Models Based on Conservation Laws Systems and Applications to Fluid Dynamics

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Key words : nonlinear conservation laws systems, fluid dynamics, physical kinetics, Boltzmann, Smoluchowski, Maxwell-Vlasov equations, approximate methods convergence.

Abstract. The paper is devoted to the background of correctness for systems of nonlinear equations possessing applied significance in mathematical physics particularly in gas and fluid dynamics (Euler equations), in physical kinetics (Boltzmann and Smoluchowski equations, Maxwell-Vlasov equations for media with discontinuity of parameters) and in phase transition models. The nonlinear operators in above equations are not continuous in Banach spaces specific for these conservation laws. There are discussed general mathematical structures, connected with approximate methods convergence. The existence and uniqueness theorems for global solutions of Cauchy problem for quasilinear and semilinear systems are proved. The problems of computations in above models are discussed too.

The mathematical models of physical systems, consisting of statistically plenty of particles (fluid dynamics, dispersible systems, plasma, systems with phase transition surfaces) and continuous mechanics models are based on fundamental relations of the balance which general name is *conservation laws*. The significant quantity of modern research on the conservation laws theory is connected with questions of correctness problem for nonlinear systems of differential and integrodifferential equations

$$\partial_t u^{(\omega)}(x,t) + \sum_{j=1}^{n} \partial_{x_j} f_j^{(\omega)}(u,x,t) = S^{(\omega)}(u,x,t) \qquad (1)$$

$$x \in \mathbf{R}_n, \quad t > 0, \quad \omega \in \Omega, \quad u\big|_{t=0} = u_0,$$

where $u=\{u^{(\omega)}\}$ is unknown vector-function, the kind of flows f_j and source (collision operator) S are considered as given by character of simulated physical process, $x \in \mathbf{R}_n$ are space coordinates, t is the time, ω are parameters, numbering equations (the set ω has arbitrary nature, for example, it may be integer numbers, real numbers etc.). Below we shall name such systems of equations as *conservation laws systems*. Their applications are well-known, particularly, in connection with gas dynamics equations and hydrodynamics, physical kinetics equations by Boltzmann and Smoluchowski, plasma theory, models of crystal growth etc. [1].

Along with correctness in a circle of problems for the conservation laws (1) traditionally the special role are played by such problems of nonlinear mathematical physics as the background of passage to the limit and asymptotics for small parameters of approximate methods, used during search of unknown solution. The complete global correctness theory for the scalar conservation law (card Ω =1) was established by S.N. Kruzhkov [2, 3]. Until recently the difficulties connected with passage to the limit in nonlinear (quasilinear and semilinear) systems of the conservation laws (1) for many methods seemed insuperable.

The extension of the concept of a solution (functional solutions) [1, 4--10] makes it possible to justify the convergence of approximate methods in presence of an a'priori estimate of approximations in $L_1^{loc}(Q,\nu)$, which is uniform in the parameter provided nonlinear operators are not continuous in this space.

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Supported by RFBR grants 02-01-01014a, 02-01-81012a

The general idea is to obtain reasonable background for computations of solutions for equation (1) which is describing the movement of great number interacting particles whose behavior is very complicated. From this point of view the local characteristics based on vector of variables u (gas density, impulse density etc.) are usually irregular functions especially in presence of turbulence. In the last case the main role are playing mean values of physical variables related to the state-space-time volumes V in $\Omega \times \mathbf{R}_n \times \mathbf{R}_1$, namely

$$\int_{\Omega \times \mathbf{R}_n \times \mathbf{R}_1} I_V u^{(\omega)}(x,t)\, \mu \otimes d_x \otimes d_t$$

where μ is the measure on particles states Ω and d_x, d_t are the Lebesgue measures on space-time variables and I_V is indicator function for volume V. The values of above integrals (functionals) for different indicator functions I_V make up the *functional solution* which exact definition is below. So the functional solutions concept is devoted to background of global correctness of Cauchy problem for equation (1) and global convergence of approximate methods to unique solution is proved provided weak approximation and weak stability take place for given approximate method $\mathfrak{AM}$

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