Bremsstrahlung Account in Photon Transport

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The new TTBIAS (Thick-Target Bremsstrahlung model Including Angular and Spatial distribution) model of bremsstrahlung account in photon transport problems in 'thick' domains is suggested. This model assumes local absorption of electrons and positrons at the point of their generation and emission of all bremsstrahlung photons generated by them. The number of bremsstrahlung photons and the spectral, angular and spatial distribution of the bremsstrahlung photon energy per one event of a primary photon interaction is calculated beforehand with the ELISA code /1/ and allocated in the tables. From the viewpoint of the photon transport equation, the offered model reduces to addition of one more integral term to the right-hand side of this equation, so the photon transport equation becomes the following

$$\begin{aligned}
&\frac{\partial f\left(\vec{r},\vec{E},t\right)}{\partial t}+\vec{v}\,\frac{\partial f\left(\vec{r},\vec{E},t\right)}{\partial \vec{r}}+\sigma\left(\vec{r},E\right)v\left(E\right)f\left(\vec{r},\vec{E},t\right)= \\
&=\iint f\left(\vec{r}',\vec{E}',t\right)\sigma\left(\vec{r}',E'\right)v\left(E'\right)N_{brem}\left(E'\right)\hat{K}_{brem}\left(\vec{r}',\vec{E}'\to\vec{r},\vec{E}\right)d\vec{r}'d\vec{E}'+ \\
&+\int f\left(\vec{r},\vec{E}',t\right)\sigma\left(\vec{r},E'\right)v\left(E'\right)K\left(\vec{r};\vec{E}'\to\vec{E}\right)d\vec{E}'+g\left(\vec{r},\vec{E},t\right).
\end{aligned} \tag{1}$$

Here $f\left(\vec{r},\vec{E},t\right)$ is the distribution function of photons at time t at a space point $\vec{r}$ over energies E and directions $\vec{\omega}$; $\vec{E}=E\cdot\vec{\omega}$; $\vec{v}=v\cdot\vec{\omega}$; v is a velocity; $\sigma\left(\vec{r},E\right)$ is a total interaction cross section; $K\left(\vec{r};\vec{E}'\to\vec{E}\right)$ is a transition kernel; $g\left(\vec{r},\vec{E},t\right)$ is a source of photons; $N_{brem}\left(E'\right)$ is a number of bremsstrahlung photons generated by photon of energy E'; $\hat{K}_{brem}\left(\vec{r}',\vec{E}'\to\vec{r},\vec{E}\right)$ is the spectral, angular, and spatial distribution of the bremsstrahlung photon per one event of a photon interaction (further, a primary photon interaction) at a point $\vec{r}'$ with energy E' and direction $\vec{\omega}'$.

The appearance of one more integral term in the right-hand side of photon transport equation (1) can be treated as introduction of a new photon interaction process, bremsstrahlung production process, to the mathematical model of the photon transport. This process is of artificial (unphysical) character. It takes place in each event of the photon interaction with any from real interaction processes (incoherent and coherent scattering, photoelectric absorption, pair and triplet production). Its total cross-section is the primary photon cross-section, the number of emitted bremsstrahlung photons is $N_{brem}\left(E'\right)$, and the differential cross-section is $\hat{K}_{brem}\left(\vec{r}',\vec{E}'\to\vec{r},\vec{E}\right)$.

The spectral, angular, and spatial distribution of the bremsstrahlung photon energy $E\cdot\hat{K}_{brem}\left(\vec{r}',\vec{E}'\to\vec{r},\vec{E}\right)$ (the distribution of the bremsstrahlung photon energy is interpolated better in terms of energy) is calculated in advance with the ELISA by solving the system of linear time-dependent electron and positron transport equations with the Monte Carlo methods in the infinite homogeneous medium with source

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$$g^{\ell}\left(\vec{r},\vec{E},t;\vec{E}_0\right)=K^{\gamma\ell}\left(\vec{r};\vec{E}_0\to\vec{E}\right)\delta\left(\vec{r}\right)\ \ \delta\left(t\right):$$

$$\begin{aligned}&\frac{\partial\ \ f^{\ell}\left(\vec{r},\vec{E},t;\vec{E}_0\right)}{\partial\ \ t}+\vec{v}^{\ell}\frac{\partial\ \ f^{\ell}\left(\vec{r},\vec{E},t;\vec{E}_0\right)}{\partial\ \ \vec{r}}+\sigma^{\ell}\left(\vec{r},E\right)v^{\ell}\left(E\right)f^{\ell}\left(\vec{r},\vec{E},t;\vec{E}_0\right)=\\&=\int\sum_{\ell'}f^{\ell'}\left(\vec{r},\vec{E}',t;\vec{E}_0\right)\sigma^{\ell'}\left(\vec{r},E'\right)v^{\ell'}\left(E'\right)K^{\ell'\ell}\left(\vec{r};\vec{E}'\to\vec{E}\right)d\vec{E}'+g^{\ell}\left(\vec{r},\vec{E},t;\vec{E}_0\right).\end{aligned}\tag{2}$$

Here $\vec{E}_0$ is the primary photon energy and direction, ℓ is a particle type (electron or positron).

Functionals $N_{brem}\left(E_0\right)$ and $\hat{K}_{brem}\left(\vec{r}_0,\vec{E}_0\to\vec{r},\vec{E}\right)\equiv K_{brem}\left(\vec{r}-\vec{r}_0;\vec{E}_0\to\vec{E}\right)$ (by virtue of homogeneity of the medium!) are expressed through the solution $f^{\ell}\left(\vec{r}-\vec{r}_0,\vec{E}',t;\vec{E}_0\right)$ to system (2) as follows:

$$\tilde{K}_{brem}\left(\vec{r}-\vec{r}_0;\vec{E}_0\to\vec{E}\right)=\iint\sum_{\ell}f^{\ell}\left(\vec{r}-\vec{r}_0,\vec{E}',t;\vec{E}_0\right)\sigma^{\ell}\left(\vec{r}-\vec{r}_0,E'\right)v^{\ell}\left(E'\right)K^{\ell,\,brem}\left(\vec{r}-\vec{r}_0;\vec{E}'\to\vec{E}\right)d\vec{E}'dt$$

$$N_{brem}\left(E_0\right)=N_{brem}\left(\vec{E}_0\right)=\iint\tilde{K}_{brem}\left(\vec{r}'';\vec{E}_0\to\vec{E}\right)\ \ d\vec{r}''d\vec{E}\ ,$$

$$\hat{K}_{brem}\left(\vec{r}_0,\vec{E}_0\to\vec{r},\vec{E}\right)\equiv\tilde{K}_{brem}\left(\vec{r}-\vec{r}_0;\vec{E}_0\to\vec{E}\right)/N_{brem}\left(E_0\right)\ ,$$

$K^{\ell,\,brem}\left(\vec{r}-\vec{r}_0;\vec{E}'\to\vec{E}\right)$ is the transition kernel in the charged particle/material interaction producing bremsstrahlung photons, $\vec{r}''=\vec{r}-\vec{r}_0$.

It was possible to describe the spectral, angular, and spatial distribution of the bremsstrahlung photon energy as function of three variables: energy *E*, cosine of angle μ between the bremsstrahlung photon direction and the primary photon direction, and distance *z* along the primary photon direction from the point of the primary photon interaction up to the point of the bremsstrahlung photon ejection (longitudinal shift). It was shown that the cross shift (the distance up to the straight line passing through a point of the primary photon interaction along the primary photon direction) can be neglected, and the longitudinal shift is better to set in dimensionless units (ratio of longitudinal shift to the full residual CDSA range of electron of maximal energy). To make the tables shorter, the partial factorization of the spectral, angular, and spatial distribution of the bremsstrahlung photon energy was done: the grid in *z* was integrated and, in each cell of the grid, the spectral, angular, and spatial distribution was represented as a product of the spectral and angular distribution and of the spatial distribution.

To increase the accuracy of calculations, the TTBIAS model has been supplemented with a model that includes the spatial distribution of annihilation photon generation points. In this model the spatial distribution of the annihilation photon generation points is calculated in advance with ELISA in cylindrical system of coordinates and is tabulated much like the spatial distribution of the bremsstrahlung photon generation points is tabulated in the TTBIAS model (only longitudinal shift). From the standpoint of the photon transport equation, the model implementation means updating the transition kernel for the pair and triplet production in the second integral term of equation (1)

$$\begin{aligned}&\tilde{K}_{pair}\left(\vec{r}',\vec{E}'\to\vec{r},\vec{E}\right)=\tilde{K}_{ann}\left(\vec{r}'\to\vec{r};\vec{E}'\right)\left\{K'_{pair}\left(\vec{r};\vec{E}'\to\vec{E}\right)+K''_{pair}\left(\vec{r};\vec{E}'\to\vec{E}\right)\right\},\\&K''_{pair}\left(\vec{r};\vec{E}'\to\vec{E}\right)=K'_{pair}\left(\vec{r};\vec{E}'\to-\vec{E}\right),\\&K'_{pair}\left(\vec{r};\vec{E}'\to\vec{E}\right)=\frac{1}{4\pi}\delta\left(E-mc^2\right).\end{aligned}\tag{3}$$

Here $\tilde{K}_{ann}\left(\vec{r}'\to\vec{r};\vec{E}'\right)$ is the spatial distribution of the annihilation photon generation points, $K'_{pair}\left(\vec{r};\vec{E}'\to\vec{E}\right)$ and $K''_{pair}\left(\vec{r};\vec{E}'\to\vec{E}\right)$ are the spectral and angular distributions of the generation of two annihilation photons of energy $E=mc^2$ each (*m* is the electron rest mass, *c* is the speed of light), moving in opposite randomly oriented directions. With taking into account (3), equation (1) becomes

$$\frac{\partial f\left(\vec{r},\vec{E},t\right)}{\partial t}+\vec{v}\frac{\partial f\left(\vec{r},\vec{E},t\right)}{\partial \vec{r}}+\sigma\left(\vec{r},E\right)v\left(E\right)f\left(\vec{r},\vec{E},t\right)=$$

$$=\iint f\left(\vec{r}',\vec{E}',t\right)\sigma\left(\vec{r}',E'\right)v\left(E'\right)N_{brem}\left(E'\right)\hat{K}_{brem}\left(\vec{r}',\vec{E}'\to\vec{r},\vec{E}\right)d\vec{r}'d\vec{E}'+$$

$$+\iint f\left(\vec{r}',\vec{E}',t\right)\sigma_{pair}\left(\vec{r}',E'\right)v\left(E'\right)\tilde{K}_{pair}\left(\vec{r}',\vec{E}'\to\vec{r},\vec{E}\right)d\vec{r}'d\vec{E}'+$$

$$+\int f\left(\vec{r},\vec{E}',t\right)\sigma_{w/o\ pair}\left(\vec{r},E'\right)v\left(E'\right)K_{w/o\ pair}\left(\vec{r};\vec{E}'\to\vec{E}\right)d\vec{E}'+g\left(\vec{r},\vec{E},t\right),$$

where

$$\sigma_{w/o\ pair}\left(\vec{r},E'\right)=\sigma_{comp}\left(\vec{r},E'\right)+\sigma_{rayl}\left(\vec{r},E'\right)+\sigma_{phot}\left(\vec{r},E'\right),$$

$$K_{w/o\ pair}\left(\vec{r};\vec{E}'\to\vec{E}\right)=\frac{\sigma_{comp}\left(\vec{r},E'\right)}{\sigma_{w/o\ pair}\left(\vec{r},E'\right)}K_{comp}\left(\vec{r};\vec{E}'\to\vec{E}\right)+$$

$$+\frac{\sigma_{rayl}\left(\vec{r},E'\right)}{\sigma_{w/o\ pair}\left(\vec{r},E'\right)}K_{rayl}\left(\vec{r};\vec{E}'\to\vec{E}\right)+\frac{\sigma_{phot}\left(\vec{r},E'\right)}{\sigma_{w/o\ pair}\left(\vec{r},E'\right)}K_{phot}\left(\vec{r};\vec{E}'\to\vec{E}\right),$$

and the subscripts *comp*, *rayl* and *phot* refer to incoherent (Compton) scattering, coherent (Rayleigh) scattering, and photoelectric absorption, respectively.

The main distinction of the given model from the TTB model (Thick-Target Bremsstrahlung model) used in the Los Alamos MCNP code /2/ is the correct account for the angular distribution of emitted bremsstrahlung photons and spatial distribution of their generation points: the TTB model assumes that all bremsstrahlung photons are emitted at the electron or positron absorption point and move along the primary photon direction. The spatial distribution of the annihilation photon generation points is not taken into account in MCNP as well.

Comparative calculations of photon transport in thick leaden layers have been performed. A task with using the TTBIAS model has been shown to run several thousands times faster then the task of the combined photon, electron and positron transport, but a few times slower then the task of photon transport without inclusion of bremsstrahlung. By the example of a leaden layer it has been found that photon releases from thick layers calculated with the TTBIAS model are in a good agreement with the transmitted photon values calculated with account for electron and positron transport. This good agreement begins to be broken only when the layer thickness becomes less than the equilibrium thickness equal to the full residual CDSA range of maximum-energy electron.

Comparison between the results of the ELISA calculations with use of the TTBIAS model and those of the MCNP calculations with use of the TTB model shows that, in this case, the MCNP calculations give the photon releases approximately 2 times higher and don't conserve average energy. The difference obtained is a result of two competing factors: underestimation of the photon output due to neglect of the spatial distribution of the bremsstrahlung and annihilation photon generation points and overestimation of the photon output due to neglect of the angular distribution of emitted bremsstrahlung photons. The backscattered photon values are therewith 1.5 – 2 times lower, and the backscattered photon energy values are 2 – 4 times lower.

The TTBIAS model has been used to calculate the task of the image of a leaden spherical shell irradiated by photon flux. It is shown that in formation of the image the basic role is played by a bremsstrahlung component essentially lowering contrast of the image.

The tables of the spectral, angular, and spatial distribution of the bremsstrahlung photon energy that constitute a key element of the TTBIAS model can be calculated only on modern multiprocessor computer systems. For this purpose the parallel version of the ELISA code has been developed basing on algorithms described in /3/. The parallelization was by particle paths, so that each processor calculated its share of the paths with its own set of random numbers. Efficiency of the parallelization algorithms used is close to 100%. Also, the tasks of photon transport with use of the TTBIAS model were calculated in the parallel mode.

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