Mathematical Modeling of the Multidimensional Radiation Transfer Problems with Parallel Computing

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Space research is such area of fundamental and applied studies which can not but be developed without computer means from the first steps of its origination. The exploration of space served as an important factor of computers improvement and of formation of new scientific directions concerning mathematical modeling of Earth's radiation field, theory of imagery transfer, theory of vision, theory of processing and pattern recognition, etc. To facilitate information and mathematical techniques is a compulsory and composite part of any space project. New perspective opportunities of mathematical modeling of atmospheric radiation of the Earth on the scales of the entire planet are connected with elaboration of soft-ware tools for the wide region of applications using super-computers with parallel architecture.

Within the framework of the development of computing means are considered the following multidimensional spherical and planar models of the radiation transfer in the atmosphere-surface system (SAS). For the basis the numerical solution of a boundary value problem (BVP) for the stationary equation of the monochromatic or quasi-monochromatic radiation transfer in the scattering, absorpting, emitting, refracting atmosphere (spherical shell or planar layer) compound spatial structure limited by the inhomogeneous reflecting underlying surface , a role by which one can play a surface (land, ocean), upper boundary of a cloud cover or hydrometeors, is accepted.

Model I. The spectral, spatial and angular structures of the atmospheric radiation under the known conditions of the illumination and the observation are calculated as the BVP solution. The spectral and spatial distributions of the integral on the angles characteristics of a radiation field (density, flows, diffusive and asymmetry coefficients etc.) are determined as the intensity functionals.

Model II. The spectral and spatial structures of the integral characteristics of atmospheric radiation are calculated as the the solutions of the problems answering to the (mathematically) exact or a miscellaneous degree proximity linear and non-linear models, which one are received from the integro-differential transfer equation by the expansions of the solution on the spherical harmonics. Models II represent also the concern for the development of the methods of the acceleration of the iteration convergence in model I.

The activities with the spherical multidimensional models of SAS are started in the 60-th years in connection with the space exploration and the development of an astrophysics and physics of planets. These models rather well mirror the main features and mechanisms of the physical process in a set of the problems.

1. The research of the radiation field in the atmosphere of a spherical planet illuminated by a parallel flux of the solar radiation. This problem has the different appendices to technical problems. At the same time it is the classic problem of an astrophysics and atmospheric optics.
2. The definition of the radiation field created by a point source in an inhomogeneous spherical shell. It not only applied, but also the problem of the classic transfer theory connected with the calculation of an influence function (Green function) BVP.

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3. The research of the reflecting properties of a full-sphere, on which one drops a parallel or diffusive external radiation flux. Such full-sphere can correspond to a model of a separate cumulus or optically of dense alternate corpuscle of a muddy medium.
4. The research of a radiation field inside a spherical cavity enclosed by a spherical shell of the matter, on which one drops an external radiation flux. This problem from the theory of the protection of a space vehicle from the radiation.

It is possible to select following types of the radiation problems requiring of the registration of a surface. The first type are the problems of an power energetics and radiation balance of the Earth, when the source is served by the sunlight radiation. Such problems are decided predominantly in an approximation of the planar model of an earth shell with the implicit or explicit registration of the contribution of the uniform lambertian or unorthotropic underlying surface. The second type are the problems of the remote sensing of the atmosphere and cloud cover, when the surface is a interference. The third type are the problems of the remote sensing of the earth surface, when it is necessary to remove (to conduct atmospheric correction) or authentically to take into account of the atmosphere influence.

The idea of our approach, called as the method of the influence functions (IF) and the spatial-frequency characteristics (SFC) is in a representation of any BVP solution by special functionals called as the optical transfer operators (OTO). The atmospheric channel is considered as a unit of an optical system of the radiation transfer and is formulated the theory of the optical transfer operator, using the mathematical vehicle of a linearly-system approach (integral of a superposition). The author has obtained the fundamental results in OTO theory : from the unified methodical bases is formulated OTO for all variety of the angular and spatial structures both the characteristics of the sources and boundaries; all non-linear approximations are submitted through linear IF and SFC; the full set of the IF and SFS basis models , necessary and sufficient for the description of the transmission characteristics of a radiation transfer system is determined; OTO is stated strictly mathematically and physically correctly in the context of a classic linearly-system approach; the theory OTO and the IF and SFC method are generalized on the two- media systems with an inner dividing boundary; the OTO theory and the IF and SFC method are formulated for the polarization problems. The explained OTO model describes the known theoretic results.

As has shown the analysis of a position of a problem of the registration and remote sensing of a surface all diversity of the approaches is reduced to three basic. First the implicit way of the registration of a refraction surface has appeared. Second is an explicit way by the IF and SFC method. Third is a method of the value function and adjoint equations (Marchuk G.I.). The IF term integrates all types of a singularity and diffusivity characteristics of the sources and boundaries.

From the unified methodical positions the four classes of the problems are considered: with horizontally homogeneous and inhomogeneous lambertian and anisotropic boundary conditions. Instead of the initial model - general BVP is proposed the new model, asymptotically exact and adequately describing of the physical process.

Asymptotically exact models are obtained by the stringent methods. As a result of the direct asymptotic expansion the solution of the initial insoluble BVP is searched as a fast converging ascending power series on the parameter and is reduced to an successive approximations procedure on a multiplicity of the radiation interaction with the boundary by a recurrent system, solvable of the influence functions method. The influence functions are calculated by the Fourier transformation method. In the computing schedule the general BVP solution was reduced to the solution of the one-dimensional complex BVP for SFC, calculation of the scene on boundary by a recurrent procedure and the functional - the solution in the preselected points of a phase volume of the initial problem. The selection of a way of the calculation of the functionals (through IF or SFC) depends on the properties of the functions describing of the boundaries and sources characteristics.

New result in the proposed approach are an reducing of the solution of the initial BVP with the difficult non-linear dependence from the properties of the boundaries to the BVP solution with the "vacuum" boundary conditions and a formulation of the vectorial linear and non-linear functionals with kernels - IF or SFC tensors. The universal functions, invariant concerning the characteristics of a polarization condition, the horizontal variations both angular relations of the boundary conditions and the BVP sources are chosen. Having a set of such invariant functions with the help of the Neumann series it is possible to receive the BVP solution with the different particular spatial and angular structure of the sources and the kernels of the reflection (transmission) operators in any approximations on a multiplicity of the interaction in view of the repeated scattering and the polarization in a medium by means of IF tensors at each interaction of the radiation with boundary. The obtained statement of the operatoric recurrence between the terms of the Neumann series member increases the efficiency of evaluations of the non-linear approximations. Instead of the registration of the reflection on each iteration on a multiplicity of the scattering in new model the calculations of the contributions of the scattering and the reflection are decomposed. It is effective, as these mechanisms have the different order.

In the problems of the radiation correction at the remote sensing of the objects and Earth surface, in the processing of the optical information, in the theories of vision and image transferring through the muddy media, in the theoretically computational bases of the designing of the optical-electronic observation systems the widespread occurrence has received an approximation of the linear systems. The linear systems are responded to the first boundary value problems of the transfer theory (with " vacuum " boundaries).

The proposed solution of a boundary value problem for a transfer equation is given by an iteration method of characteristics. Since the majority of photons does not give any noticeable contribution to collisions of the higher order than the first, no need appears to consider multiple scattering on the level of the radiation intensity registration. This fact explains the possibility to eliminate the relevant effects of scattering on the specified spherical and planar subregions. Serves as an analogue of known infrequent grids method, the proposed improvements enable to speed up the convergence procedures in the related iterative process. In essence, these improvements are very close to the relaxation method and non-linear methods of speeding up the convergence of solutions on subregions when a high optical non-uniformity of space is typical.

The inversion of the differential operator in the partial derivatives plays the role of the integration along the characteristics on each iteration. The net-characteristic method is to be utilized for calculation of a complete set of grid values of intensity under any forms of initial grid source functions.

The net-characteristic approach is grounded on the analysis of an equation of a ray path in space in view of the first integrals of the differential transfer operator. Such general-purpose approach has allowed to state the combined planar-spherical model (spherical shell with planar subregions).

The numerical model of the radiative transfer in a system atmosphere-ocean (SAO) is considered. The mechanism of the formation of the radiation fields in SAO are researched in view of the short-wave radiation interchange between the media.

The writers develop the one-dimensional models of the solar radiation transfer in SAO on the basis of two approaches. In the one model the calculation implements an the iteration method of the characteristics for two-media SAO This model by the capabilities is close to the foreign models. In the second model the radiation SAO is received with the help of the optical transfer operator through the influence functions of atmosphere (IFA) and ocean (IFO). This new original approach is formulated in the context of a classic linearly-system approach.

The problems of the IFA and IFO definition coincide the customary one-dimensional problem of the radiative transfer theory in a planar layer illuminated by an external parallel flux. The method of the solution of these problems depends on the optical-physical characteristics of media. The atmosphere can be cloudy or cloudless. The ocean is possible consider as a finite or semi-infinite layer.

The influence functions method as a method of the solution of the boundary value problems in the radiative transfer theory in the environments is specially effective in conditions of the very mutable atmosphere and more conservative ocean. This method has general-purpose property: it is possible to integrate the homogeneous and inhomogeneous finite and semi-infinite layers in unified SAO. Through IFA and IFO it is possible to estimate a backscatter in the atmosphere and ocean, and also to receive a reflection coefficient of a surface of water basin in view of a multiscattering radiation in a water. Having the IFA and IFO set, it is possible variously to structure a radiation field in SAO and in detail to research the mechanisms of the forming of the atmospheric radiation under the ocean influence .

The constructed base of variety of mathematical models using the vector IFs and the vector OTO values can allow to apply the proposed new algorithms of numerical modeling of the polarized optical and millimeter (in a quasi-optical approximation) radiative transfer in the systems "atmosphere-land", "atmosphere-ocean", "atmosphere-cloud", "atmosphere-hydrometeors", "atmosphere-vegetative community". Besides that, the radiation correction procedures are outlined in the related methods of remote sensing applications. Additional applications are concerned the theory of vision and the transfer theory of images through any opaque polarizing media.

The algorithms of parallel calculations are given by:

1. according to physical models:
- Multispectral;
- Optical and geophysical weather (using specified coefficients);
- In accordance with the relevant sources.

2. methodological parallelization for boundary condition problems:
- On parameters of influence functions;
- On parameters of spatial and frequency characteristics;
- On the mathematical functionals used.

3. algorithmic techniques for a multi-dimensional spherical model:
- Single scattering along the characteristics;
- Multiple scattering along the integrals of collisions;
- On sub-areas with different media.

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