Economical 3D Hydrodynamics Model for Homogeneous Water Basins and its Parallel Realization

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1. Continued model

High precision 3D hydrodynamics model for shallow water basins has been built and applied earlier [1]. However, model examined below is of satisfactory accuracy in a number of cases. This model may be considered as the simplest and economical 3D hydrodynamics model for shallow water basins. Present model describes water medium motion in basins where density gradient is neglible and acceleration of motion in vertical direction is neglected. To obtain the model we use following system of equations:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-lv=-\frac{1}{\rho_0}\frac{\partial p}{\partial x}+\frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right)+A_l\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right); \tag{1}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+lu=-\frac{1}{\rho_0}\frac{\partial p}{\partial y}+\frac{\partial}{\partial z}\left(\nu\frac{\partial v}{\partial z}\right)+A_l\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right); \tag{2}$$

$$\frac{\partial p}{\partial z}=\rho_0 g; \tag{3}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0. \tag{4}$$

Let us consider $\xi=\xi(x,y,t)$ - elevation function. It shows free sea surface departure from undisturbed state on *Oz* axis, directed vertically down. Integrating hydrostatics equation (3) vertically from -ξ till *z* coordinate, taking into account that atmospheric pressure on undisturbed sea surface level is constant $\boldsymbol{p_a}$, we obtain

$$p=p_a+\rho_0 g\int_{-\xi}^{z} dz=p_a+\rho_0 g\xi+\rho_0 gz \tag{5}$$

Using equation (5), we find partial derivatives $\frac{\partial p}{\partial x}=\rho_0 g\frac{\partial \xi}{\partial x}$, $\frac{\partial p}{\partial y}=\rho_0 g\frac{\partial \xi}{\partial y}$ and if we substitute them into equations (1), (2) we obtain:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-lv=-g\frac{\partial \xi}{\partial x}+\frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right)+A_l\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right); \tag{6}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+lu=-g\frac{\partial \xi}{\partial y}+\frac{\partial}{\partial z}\left(\nu\frac{\partial v}{\partial z}\right)+A_l\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right). \tag{7}$$

We integrate equation of continuity (4) on *z* from -ξ till *H* and obtain expression

$$w\big|_{z=H}-w\big|_{z=-\xi}=-\int_{-\xi}^{H}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)dz. \tag{8}$$

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Using equality

$$\frac{\partial}{\partial x}\left(\int_{-\xi}^{H} udz\right)+\frac{\partial}{\partial y}\left(\int_{-\xi}^{H} vdz\right)=u\Big|_{z=H}*\frac{\partial H}{\partial x}+u\Big|_{z=-\xi}*\frac{\partial \xi}{\partial x}+\int_{-\xi}^{H}\frac{\partial u}{\partial x}dz+v\Big|_{z=H}*\frac{\partial H}{\partial y}+v\Big|_{z=-\xi}*\frac{\partial \xi}{\partial y}+\int_{-\xi}^{H}\frac{\partial v}{\partial y}dz,$$

and boundary conditions at the bottom $u=v=0,\ z=H$, and kinematic conditions at the free surface $w\Big|_{z=-\xi}=-\left(\frac{\partial \xi}{\partial t}+u\Big|_{z=-\xi}*\frac{\partial \xi}{\partial x}+v\Big|_{z=-\xi}*\frac{\partial \xi}{\partial y}\right)$ we have

$$\frac{\partial}{\partial x}\left(\int_{-\xi}^{H} udz\right)+\frac{\partial}{\partial y}\left(\int_{-\xi}^{H} vdz\right)=w\Big|_{z=H}-w\Big|_{z=-\xi}-\frac{\partial \xi}{\partial t}+\int_{-\xi}^{H}\frac{\partial u}{\partial x}dz+\int_{-\xi}^{H}\frac{\partial v}{\partial y}dz\ .$$

Considering last equation and formula (8) we obtain equation for elevation function

$$\frac{\partial \xi}{\partial t}=-\frac{\partial}{\partial x}\left(\int_{-\xi}^{H} udz\right)-\frac{\partial}{\partial y}\left(\int_{-\xi}^{H} vdz\right). \tag{9}$$

System of equations (6), (7), (9) and (4) are being considered in some G - domain, which is closed basin, confined by undisturbed sea surface $z=0$ bottom $H=H(x,y)$ and cylindrical boundary surface σ. It is necessary to add initial conditions

$$\begin{aligned}&u(x,y,z,0)=u_0(x,y,z),\quad v(x,y,z,0)=v_0(x,y,z),\\&\xi(x,y,z,0)=\xi_0(x,y,z),\quad (x,y,z)\in\overline{G},\ t=0,\end{aligned} \tag{10}$$

Initial conditions help to reconstruct initial field for velocity vertical component

$$w(x,y,z,0)=w_0(x,y,z)\ ,\quad (x,y,z)\in\overline{G},\ t=0\ .$$

Boundary conditions are formulated as follows. For velocity horizontal components: in common case, velocity function coordinates *u* and ν must be set on fluid surface boundaries, on solid surface boundaries – adhesion conditions:

$$u=v=0, \tag{11}$$

-on free surface, using wind friction components $\tau_x,\quad \tau_y$

$$\rho_0\nu\frac{\partial u}{\partial z}=-\tau_x,\quad \rho_0\nu\frac{\partial v}{\partial z}=-\tau_y\ , \tag{12}$$

-on the bottom we use Chezi formula

$$\nu\frac{\partial u}{\partial z}=\frac{g\sqrt{u^2+v^2}}{C_z^2}u,\quad \nu\frac{\partial v}{\partial z}=\frac{g\sqrt{u^2+v^2}}{C_z^2}v, \tag{13}$$

where C_z - Chezi coefficient.

2. Splitting difference scheme.

It is possible to take into account three processes, described by the system (4), (6), (7), (9): momentum transfer along water flows trajectories, diffusion and flow fields adaptation. Diffusion and transfer processes in horizontal and vertical directions for shallow water basins, with ratio $L/H\approx 10^4$ and more, where *L* and *H* are characteristic dimensions of water basin correspondingly in horizontal directions and vertical, have various execution time “scales”. Proceeding from above stated physical prerequisites we write differential equations splitting system. This system corresponds to initial system. We have following systems, sequentially solved on time interval $t_n < t \le t_{n+1}$:

$$\frac{1}{4}\frac{\partial u}{\partial t}+u^n\frac{\partial u}{\partial x}+v^n\frac{\partial u}{\partial y}=0,\ \frac{1}{4}\frac{\partial u}{\partial t}+w^n\frac{\partial u}{\partial z}=0,\ \frac{1}{4}\frac{\partial v}{\partial t}+u^n\frac{\partial v}{\partial x}+v^n\frac{\partial v}{\partial y}=0,\ \frac{1}{4}\frac{\partial v}{\partial t}+w^n\frac{\partial v}{\partial z}=0, \tag{14}$$

$$\frac{1}{4}\frac{\partial u}{\partial t}=A_l\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right),\qquad \frac{1}{4}\frac{\partial v}{\partial t}=A_l\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \tag{15}$$

$$\frac{1}{4}\frac{\partial u}{\partial t}=\frac{\partial}{\partial z}\left(\nu\frac{\partial u}{\partial z}\right)-g\frac{\partial\xi}{\partial x}+lv,\quad \frac{1}{4}\frac{\partial v}{\partial t}=\frac{\partial}{\partial z}\left(\nu\frac{\partial v}{\partial z}\right)-g\frac{\partial\xi}{\partial y}-lu, \tag{16}$$

$$\frac{\partial\xi}{\partial t}=-\frac{\partial}{\partial x}\left(\int_{-\xi}^{H}udz\right)-\frac{\partial}{\partial y}\left(\int_{-\xi}^{H}vdz\right), \tag{17}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0\ . \tag{18}$$

Symbol “*n*” means that linearization has been done in transfer equations – velocity components values in equations (14) are taken from previous time step.

Space grid, where system (14)-(18) discretization will be executed, consists of rectangular parallelepipeds with dimensions h_x x h_y x h_{zk}, where h_x, h_y are steps of plane grid, which is uniform in horizontal directions; h_{zk} is step of the grid, which is nonuniform in *Oz* direction in common case. We give number to each cell of the space grid – for rectangular parallelepiped - three indexes *i, j, k*, referring them to parallelepiped center of symmetry. Thicknesses of bottom parallelepipeds – cells with the numbers *(i, j, m)*, depend on numbers *(i, j)*, and thicknesses of surface parallelepipeds - cells with the numbers *(i, j, M)* - besides, depend on free surface location and, therefore, time. Grid functions of velocity components $u_{i+1/2,j,k}$, $v_{i,j+1/2,k}$, $w_{i,j,k+1/2}$ are defined in the centers of symmetry of parallelepiped verges *i, j, k*, but elevation function $\xi_{i,j}$ is defined in the point of disturbed surface, located above parallelepiped center of symmetry *i, j, k*.

We build "semi emplicit " approximation. For this purpose we approximate equations (14), using the upwind explicit schemes. For the equations (15)-(17) we use implicit difference approximation.

We write out approximation of the equations (16) as solved regarding functions $u^{n+1}_{i+1/2,j,k}$ and $v^{n+1}_{i,j+1/2,k}$:

$$u^{n+1}_{i+1/2,j,k}=\tau\frac{\nu_{k+1/2}\dfrac{u^{n+1}_{i+1/2,j,k+1}-u^{n+1}_{i+1/2,j,k}}{h_{z,k+1/2}}-\nu_{k-1/2}\dfrac{u^{n+1}_{i+1/2,j,k}-u^{n+1}_{i+1/2,j,k-1}}{h_{z,k-1/2}}}{h_{z,k}}-g\frac{\tau}{h_x}\left(\xi^{n+1}_{i+1,j}-\xi^{n+1}_{i,j}\right)+\tau lv^{n+3/4}_{i+1/2,j,k}+u^{n+3/4}_{i+1/2,j,k}, \tag{19}$$

$$u^{n+1}_{i+1/2,j,k}=\tau\frac{\nu_{k+1/2}\dfrac{v^{n+1}_{i,j+1/2,k+1}-v^{n+1}_{i,j+1/2,k}}{h_{z,k+1/2}}-\nu_{k-1/2}\dfrac{u^{n+1}_{i,j+1/2,k}-u^{n+1}_{i,j+1/2,k-1}}{h_{z,k-1/2}}}{h_{z,k}}-g\frac{\tau}{h_y}\left(\xi^{n+1}_{i,j+1}-\xi^{n+1}_{i,j}\right)-\tau lu^{n+3/4}_{i,j+1/2,k}+v^{n+3/4}_{i,j+1/2,k}, \tag{20}$$

In equations (19) and (20) the following indications were used $h_{z,k+1/2}=0.5\left(h_{z,k+1}+h_{zk}\right)$, $h_{z,k-1/2}=0.5\left(h_{z,k-1}+h_{zk}\right)$, where minimum and maximum values of index *k* - correspondingly *m=m(i,j)*+1, *M=M(i,j)* - when computing $h_{z,k-1/2}$ and *m=m(i,j), M=M(i,j)*-1 - correspondingly when computing $h_{z,k+1/2}$. We set difference boundary conditions. To make more comfort we will also consider dummy cells of 3D grid domain with numbers *m-1* and *M+1*. Then boundary conditions in vertical will be as follows

$$\nu_{M+1/2}\frac{u^{n+1}_{i+1/2,j,M+1}-u^{n+1}_{i+1/2,j,M}}{h_{z,M+1/2}}=\tau^w_x,\ \nu_{M+1/2}\frac{v^{n+1}_{i,j+1/2,M+1}-v^{n+1}_{i,j+1/2,M}}{h_{z,M+1/2}}=\tau^w_x, \tag{21}$$

$$\begin{aligned}&\nu_{m+1/2}\frac{u^{n+1}_{i+1/2,j,m}-u^{n+1}_{i+1/2,j,m-1}}{h_{z,m-1/2}}=\frac{g\sqrt{\left(u^n_{i+1/2,j,m}\right)^2+\left(v^n_{i+1/2,j,m}\right)^2}}{C_z^2}u^{n+1}_{i+1/2,j,m},\\&\nu_{m+1/2}\frac{v^{n+1}_{i,j+1/2,m}-v^{n+1}_{i,j+1/2,m-1}}{h_{z,m-1/2}}=\frac{g\sqrt{\left(u^n_{i+1/2,j,m}\right)^2+\left(v^n_{i+1/2,j,m}\right)^2}}{C_z^2}v^{n+1}_{i,j+1/2,m},\end{aligned} \tag{22}$$

Approximating equation of level surface we use formula of rectangles for approximate substitution of integrals. We have

$$\xi_{i,j}^{n+1}=\xi_{i,j}^{n}-\frac{\tau}{h_x}\left(\sum_{k=m(i+1/2,j)}^{M(i+1/2,j)} h_{zk}u_{i+1/2,j,k}^{n+1}-\sum_{k=m(i-1/2,j)}^{M(i-1/2,j)} h_{zk}u_{i-1/2,j,k}^{n+1}\right)-\frac{\tau}{h_y}\left(\sum_{k=m(i,j+1/2)}^{M(i,j+1/2)} h_{zk}v_{i,j+1/2,k}^{n+1}-\sum_{k=m(i,j-1/2)}^{M(i,j-1/2)} h_{zk}v_{i,j-1/2,k}^{n+1}\right) \quad (23)$$

Equation of continuity approximation leads to difference equations system

$$w_{i,j.k+1/2}^{n+1}=w_{i,j.k-1/2}^{n+1}-\frac{h_{z,i+1/2,j,k}^{n}u_{i+1/2,j,k}^{n+1}-h_{i-1/2,j,k}^{n}u_{i-1/2,j,k}^{n+1}}{h_x}-\frac{h_{z,i,j+1/2,j,k}^{n}u_{i,j+1/2,j,k}^{n+1}-h_{i,j-1/2,j,k}^{n}u_{i,j-1/2,j,k}^{n+1}}{h_y},\quad k=m,\dots,M \quad (24)$$

with boundary conditions $w_{i,j,m-1/2}^{n+1}=0$.

Scheme has summary approximation error $O\left(|h|+\tau\right)$, where $|h|=\sqrt{h_x^2+h_y^2+\max_k\left\{h_{zk}^2\right\}}$. Estimation of time step permissible value may be as follows

$$\tau\le\min_{i,j,k}\left\{\frac{1}{u_{i+1/2,j,k}/h_x+v_{i,j+1/2,k}/h_y},\frac{h_{zk}}{w_{i,j,k+1/2}}\right\}. \quad (25)$$

3. Scheme numerical realization.

We transform systems (19), (20) and (23), excluding from (23) velocity components . For this purpose we write equations (19) and (20) in matrix form, using boundary conditions (21), (22):

$$A_{i+1/2,j}^{n}U_{i+1/2,j}^{n+1}=F_{i+1/2,j}^{n}-g\frac{\tau}{h_x}\left(\xi_{i+1,j}^{n+1}-\xi_{i,j}^{n}\right)\Delta H_{i+1/2,j}^{n},$$

$$A_{i,j+1/2}^{n}V_{i,j+1/2}^{n+1}=F_{i,j+1/2}^{n}-g\frac{\tau}{h_y}\left(\xi_{i,j+1}^{n+1}-\xi_{i,j}^{n}\right)\Delta H_{i,j+1/2}^{n}, \quad (26)$$

where vectors and matrixes, taking place in the systems (26) are following

$$U_{i+1/2,j}^{n+1}=\begin{Vmatrix}u_{i+1/2,j,M}^{n+1}\\u_{i+1/2,j,M-1}^{n+1}\\\vdots\\u_{i+1/2,j,m}^{n+1}\end{Vmatrix},\ V_{i,j+1/2}^{n+1}=\begin{Vmatrix}v_{i,j+1/2,M}^{n+1}\\v_{i,j+1/2,M-1}^{n+1}\\\vdots\\v_{i,j+1/2,m}^{n+1}\end{Vmatrix},\ \Delta H_{i+1/2,j}^{n}=\begin{Vmatrix}h_{z,i+1/2,j,M}^{n}\\h_{z,i+1/2,j,M-1}^{n}\\\vdots\\h_{z,i+1/2,j,m}^{n}\end{Vmatrix},\ \Delta H_{i,j+1/2}^{n}=\begin{Vmatrix}h_{z,i,j+1/2,M}^{n}\\h_{z,i,j+1/2,M-1}^{n}\\\vdots\\h_{z,i,j+1/2,m}^{n}\end{Vmatrix},$$

$$F_{i+1/2,j}^{n}=\begin{Vmatrix}h_{z\,i+1/2,j,M}^{n}\left(u_{i+1/2,j,M}^{n+3/4}+\tau l v_{i+1/2,j,M}^{n+3/4}\right)+\tau\cdot\tau_x^w\\h_{z\,i+1/2,j,M-1}^{n}\left(u_{i+1/2,j,M-1}^{n+3/4}+\tau l v_{i+1/2,j,M-1}^{n+3/4}\right)\\\vdots\\h_{z\,i+1/2,j,m}^{n}\left(u_{i+1/2,j,m}^{n+3/4}+\tau l v_{i+1/2,j,m}^{n+3/4}\right)\end{Vmatrix},$$

$$G_{i,j+1/2}^{n}=\begin{Vmatrix}h_{z\,i,j+1/2,M}^{n}\left(v_{i,j+1/2,M}^{n+3/4}-\tau l u_{i,j+1/2,M}^{n+3/4}\right)+\tau\cdot\tau_y^w\\h_{z\,i,j+1/2,M-1}^{n}\left(v_{i,j+1/2,M-1}^{n+3/4}-\tau l u_{i,j+1/2,M-1}^{n+3/4}\right)\\\vdots\\h_{z\,i,j+1/2,m}^{n}\left(v_{i,j+1/2,m}^{n+3/4}-\tau l u_{i,j+1/2,m}^{n+3/4}\right)\end{Vmatrix}, \quad (27)$$

$$A^n_{i+\frac{1}{2},j}=\left(\begin{Vmatrix} h^n_{zM}+\frac{\tau\cdot\nu_{M-\frac{1}{2}}}{h^n_{zM-\frac{1}{2}}} & -\frac{\tau\cdot\nu_{M-\frac{1}{2}}}{h^n_{zM-\frac{1}{2}}} & \ldots & 0 \\ -\frac{\tau\cdot\nu_{M-\frac{1}{2}}}{h^n_{zM-\frac{1}{2}}} & h^n_{zM-1}+\frac{\tau\cdot\nu_{M-\frac{1}{2}}}{h^n_{zM-\frac{1}{2}}}+\frac{\tau\cdot\nu_{M-\frac{3}{2}}}{h^n_{zM-\frac{3}{2}}} & -\frac{\tau\cdot\nu_{M-\frac{3}{2}}}{h^n_{zM-\frac{3}{2}}}\ldots & 0 \\ . & . & . & . \\ 0 & \ldots & -\frac{\tau\cdot\nu_{m+\frac{1}{2}}}{h^n_{zm+\frac{1}{2}}} & h^n_{zm+\frac{1}{2}}+\frac{\tau\cdot\nu_{m+\frac{1}{2}}}{h^n_{zm+\frac{1}{2}}}+\frac{g\tau\sqrt{\left(u^n_m\right)^2+\left(V^n_m\right)^2}}{C_z^2} \end{Vmatrix}\right)_{i+\frac{1}{2},j}$$

Matrix $A^n_{i,j+\frac{1}{2}}$ is defined by the same way. Systems (26) are positive defined and symmetric three diagonal systems. Matrices of systems have inverse matrices. Systems (27) are realized by Thomas algorithm. Multiplying both parts of the system (27) correspondingly by $\left(A^n_{i+\frac{1}{2},j}\right)^{-1}, \left(A^n_{i,j+\frac{1}{2}}\right)^{-1}$ and substituting vectors $U^{n+1}_{i+1/2,j,k}$ and $V^{n+1}_{i,j+1/2,k}$ in to system (23), we obtain system for $\xi^{n+1}_{i,j}$:

$$\xi^{n+1}_{i,j}-g\frac{\tau^2}{h_x^2}\left(\left(\left(\Delta H\right)^T A^{-1}\Delta H\right)^n_{i+\frac{1}{2},j}\left(\xi^{n+1}_{i+1,j}-\xi^{n+1}_{i,j}\right)-\left(\left(\Delta H\right)^T A^{-1}\Delta H\right)^n_{i-\frac{1}{2},j}\left(\xi^{n+1}_{i,j}-\xi^{n+1}_{i-1,j}\right)\right)-$$

$$-g\frac{\tau^2}{h_y^2}\left(\left(\left(\Delta H\right)^T A^{-1}\Delta H\right)^n_{i,j+\frac{1}{2}}\left(\xi^{n+1}_{i,j+1}-\xi^{n+1}_{i,j}\right)-\left(\left(\Delta H\right)^T A^{-1}\Delta H\right)^n_{i,j-\frac{1}{2}}\left(\xi^{n+1}_{i,j}-\xi^{n+1}_{i,j-1}\right)\right)=$$

$$\xi^n_{i,j}-g\frac{\tau}{h_x}\left(\left(\left(\Delta H\right)^T A^{-1}F\right)^n_{i+\frac{1}{2},j}-\left(\left(\Delta H\right)^T A^{-1}F\right)^n_{i-\frac{1}{2},j}\right)$$

$$-g\frac{\tau}{h_y}\left(\left(\left(\Delta H\right)^T A^{-1}G\right)^n_{i,j+\frac{1}{2}}-\left(\left(\Delta H\right)^T A^{-1}G\right)^n_{i,j-\frac{1}{2}}\right). \quad (28)$$

It may be shown that system (28) has positive defined and symmetric matrix. For its realization it is possible to use parallel variant of ATTM [1, 2]. To compute difference equations system (27) coefficients, which are $\left(\Delta H\right)^T A^{-1}\Delta H$ and take place in the left and right parts, we apply Thomas algorithm on each time step. Systems (25) solving may be found, using parallel variant of cyclic reduction method [3].

We propose theoretical performance estimation of multiprocessor systems (MPS) in dependence of processor number *n*, t_e/t_a - ratio, where t_e - one machine word time exchange information between two processors, t_a- typical time arithmetic operation, and *N*- number of nodes of grid in each coordinate direction have been presented for various type connection between processors. This theoretical analysis allows to choose the optimal characteristic of multiprocessor system (*n* - number of processors, t_e/t_a - ratio) in dependence of N - number of nodes of grid in each coordinate direction for various connection structure between processors.

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