Application of Methods Error-free Calculations in Hydrology

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One of the important problem of modern hydrology is increasing accuracy of computation different characteristic of surface flow (wave of freshet, minimal flow). Spatial heterogeneity flow forming factors, their difficult depending on climatic and physical characteristics of pool led to necessity of using linear schemes with middling parameters for separate segments of pool in mathematical modeling [1].

For such problems of large pool are widely applied systems of linear equations with many equations. The round off errors could led to big errors and the absolutely wrong results in some cases [2,3]. There is a method to avoid such errors. It's connected with residue number system. All rationales translate into integers numbers. After decision system of linear equations by Gaussian method the results are converted back to positional representation [4,5].

Let the system of linear equations is written in the following form:

$$A\,x = B, \tag{1}$$

where x and B vectors , $x = (x_1, x_2, ..., x_n)$, $b = (b_1, b_2, ..., b_n)$, and matrix A with a_{ij} , $A = \{ a_{ij} \}$, where j = 1,2, ... , n; i = 1,2, ... n .

It's very difficult to determine process of spreading round off errors during the computation of solving system of linear equations [6]. The process has been investigated by Gaussian method of solving system of linear equations (1) and also calculated discrepancies of the solution in the following form:

$$\Delta_i = \left| \frac{\sum_{k=1}^{n} a_{ik} \tilde{x}_k - b_i}{b_i} \right|, \tag{2}$$

where $\tilde{x}_i$ - root of the system of linear equations, i = 1,2, ..., n.

The results of the program as shown in Fig. 1 denote the discrepancies in Gaussian method exceed real coordinates of vector b ten times.

The Table 1 displays that maximal errors of solving system of linear equations are extremely increasing with quantity of equations. And the solution for more than 10^3 numbers of linear equations is considerably differ from right.

There are some different iteration methods which able decrease round off errors but not completely avoids them.

There is a method to obviate the errors using residue or modulo arithmetic [4,5].

The process of unerror calculation includes:

1) Conversion all numbers from positional representation to the residue number of system. (RNS).

Table 1. Maximum Discrepancies

Number of equations	Maximum Discrepancies
10	0.000000000000154
100	0.0000000000309
500	0,0003
1000	0,308
2000	64,93

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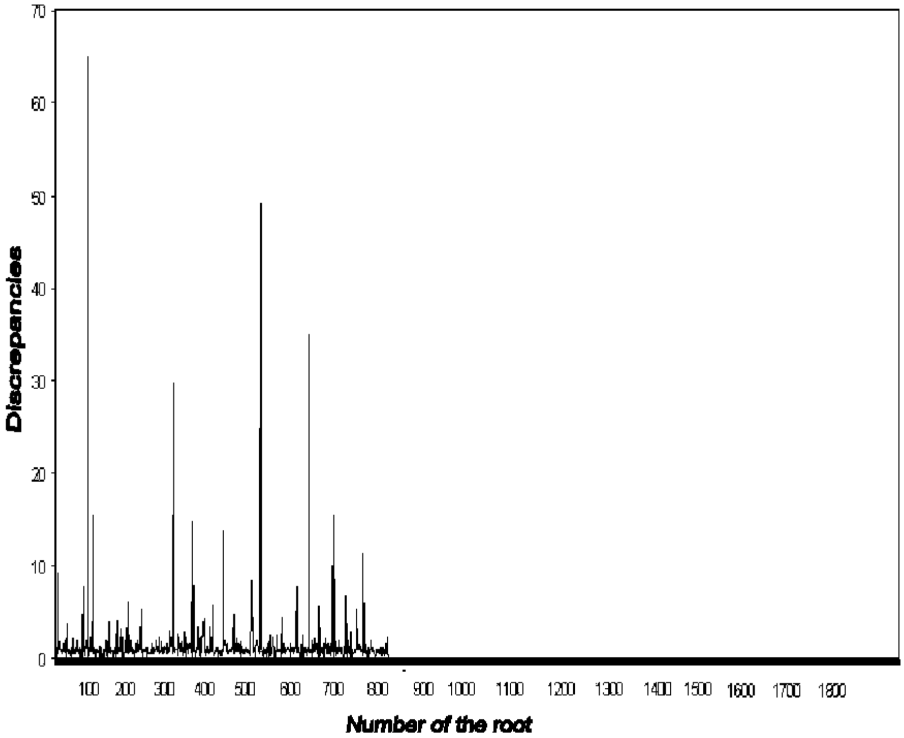

Fig 1. Discrepancies in Gaussian method

2) Unerror calculation in RNS
3) Converting results to positional number system.

Single and multiply modulus residue arithmetic has been investigated to carry out error free computation on rational numbers [4] and from the point of view computer architecture RNS algebraic operators add, subtract, multiply, divide and especially multi module RNS are convenient to implement on parallel computer systems. The single module RNS has disadvantages: operating with huge numbers during, not quick calculation with compare to PNS. Hensel code arithmetic [2] and multi module system are convenient for the parallel realization and faster than single module arithmetic. So, the table 2 displays times of solving system of linear equations in double module system, single module system and in positive number system.

Table 2. Times of solving system of linear equations.

Number of equations	METHOD OF SOLVING		
	Standard	Unerror (single module)	Unerror (double module)
	Time of solving, sec	Time of solving, sec	Time of solving, sec
10	<1	2	<1
20	<1	5	<1
30	1	20	1
40	1	40	2,5
50	1	75	4,5
60	1	122	6
70	1	130	8
80	1	300	12
90	1	440	15
100	1	556	21
150	1,5	9120	131

Test shows that computing errors increasing with the number of equations. Solution of system , consisting of more than 10^3 equations, as follows from the table maximal discrepancies, received by Gaussian method is erroneous, therefore given method cannot be used for the decision system of such orders. Using the multi module residue arithmetic provides absolutely exact decision with Gauss method. The results of the numerical experiments show, that increasing module the RNS reduces speed of computation , and using multi modular arithmetic for parallel processing on all modules provides comparable with traditional speed.

References

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