Laterally heterogeneous whole earth modeling based on the spectral Fourier-Laguerre method

Boris G. Mikhailenko*, Alexander A. Mikhailov, Galina V. Reshetova

Institute of Computational Mathematics and Mathematical Geophysics,
Siberian Branch of RAS, Novosibirsk, Russia

Introduction

A method for the simulation of seismic wave propagation in the laterally heterogeneous whole earth models is presented by solution of the elastodynamic equations in the 3D cylindrical coordinates.

The problem is simplified by considering a slice through the sphere and solving the elastodynamic equations in a 3D cylindrical coordinate system rather than in spherical coordinates.

As is shown by T. Furumura et al. (1998), the main difference in wave fields, obtained by approximation of the full spherical geometry using the 2D cylindrical coordinate system, arises from the geometrical spreading factor and from replacement of the 3D seismic sources by the 2D sources. All these factors are taken into account when we deal with the 3D cylindrical coordinate system.

In order to come to a compromise between realism and computational efficiency, we have developed a 2.5D spectral Fourier-Laguerre approach for calculation of the 3D elastic wave fields in cylindrical coordinates, when the elastic parameters of the medium depend on the radial (r) and the azimuthal (φ) coordinates and do not depend on the vertical (z) coordinate.

At the first step of solution, the original problem is split to a series of independent 2D problems by means of the finite integral Fourier transform.

The obtained in such a way 2D problems for each spatial frequency reduce to a system of algebraic equations by using the finite integral Fourier transform with respect to the azimuthal coordinate (φ), the integral Laguerre transform with respect to the temporal coordinate, and the finite difference technique of fourth order accuracy with respect to the radial coordinate.

Thus, we arrive at the system of algebraic equations with a matrix independent of the parameter j – the degree of the Laguerre polynomials. In this case, only the right-hand side of the system has the recurrent dependence on the parameter j, which is an analogue to the temporal frequency in the frequency domain methods. For solving the obtained system with a great number of the right-hand sides one can use fast methods such as the Kholetsky method. As this takes place, the matrix is only once transformed as opposed to the frequency domain forward modeling, when we have to transform a matrix for each temporal frequency, and this requires high computational costs.

However, when simulating the seismic wave propagation in the laterally - heterogeneous Earth's models for high frequencies we deal with a necessity to solve linear systems with several millions equations. In this situation we use iterative methods, such as the conjugate-gradient algorithms, since direct methods are too expensive in terms of computer memory and CPU-time requirements. In this case application of the integral Laguerre transform results in the well-conditioned matrices and provides fast convergence of iterative methods. On this stage, the proposed algorithm is readily parallelized on multi-processor computers.

* E-mail: mikh@sscc.ru

Statement of the Problem

We will consider a 3D cylindrical coordinate system with the coordinates (r,φ,z). The first order system of equations for the velocities and stresses is written down in the symmetric form:

$$\begin{aligned}
&\rho\frac{\partial u_z}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\sigma_{rz}\right)+\frac{1}{r}\frac{\partial\sigma_{\varphi z}}{\partial\varphi}+\frac{\partial\sigma_{zz}}{\partial z}+F_z\\
&\rho\frac{\partial u_r}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\sigma_{rr}\right)+\frac{1}{r}\frac{\partial\sigma_{\varphi r}}{\partial\varphi}+\frac{\partial\sigma_{zr}}{\partial z}-\frac{\sigma_{\varphi\varphi}}{r}+F_r\\
&\rho\frac{\partial u_\varphi}{\partial t}=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\sigma_{r\varphi}\right)+\frac{1}{r}\frac{\partial\sigma_{\varphi\varphi}}{\partial\varphi}+\frac{\partial\sigma_{z\varphi}}{\partial z}+F_\varphi\\
&\frac{1}{\mu}\frac{\partial\sigma_{rz}}{\partial t}=\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\\
&\frac{1}{\mu}\frac{\partial\sigma_{r\varphi}}{\partial t}=r\frac{\partial}{\partial r}\left(\frac{1}{r}u_\varphi\right)+\frac{1}{r}\frac{\partial u_r}{\partial\varphi}\\
&\frac{1}{\mu}\frac{\partial\sigma_{\varphi z}}{\partial t}=\frac{1}{r}\frac{\partial u_z}{\partial\varphi}+\frac{\partial u_\varphi}{\partial z}\\
&e_1\frac{\partial\sigma_{rr}}{\partial t}+e_2\frac{\partial\sigma_{\varphi\varphi}}{\partial t}+e_2\frac{\partial\sigma_{zz}}{\partial t}=\frac{\partial u_r}{\partial r}\\
&e_2\frac{\partial\sigma_{rr}}{\partial t}+e_2\frac{\partial\sigma_{\varphi\varphi}}{\partial t}+e_1\frac{\partial\sigma_{zz}}{\partial t}=\frac{\partial u_z}{\partial z}\\
&e_2\frac{\partial\sigma_{rr}}{\partial t}+e_1\frac{\partial\sigma_{\varphi\varphi}}{\partial t}+e_2\frac{\partial\sigma_{zz}}{\partial t}=\frac{1}{r}\left(\frac{\partial u_\varphi}{\partial\varphi}+u_r\right),
\end{aligned}\tag{1}$$

where

$$e_1=\frac{\lambda+\mu}{\mu\left(3\lambda+2\mu\right)},\quad e_2=\frac{-\lambda}{2\mu\left(3\lambda+2\mu\right)}.$$

Equation (1) will be solved with the following boundary conditions at the free surface:

$$\sigma_{rr}=\sigma_{r\varphi}=\sigma_{\varphi\varphi}=0\quad\text{at } r=a \tag{2}$$

and with zero initial data.

In equation (1), u_r,u_φ,u_z are velocity components, σ_{ij} are components of the stress tensor, $F_p=F_p\left(r,\varphi,z,t\right)$ $\left(p=r,\varphi,z\right)$ is the body force. We assume the medium to be invariant in direction z, that is, $\lambda=\lambda\left(r,\varphi\right)$, $\mu=\mu\left(r,\varphi\right)$, $\rho=\rho\left(r,\varphi\right)$.

At the first step of the solution of our problem, we apply the double finite Fourier transforms with respect to the coordinates z and φ. Let us note that the use of the first order system of equations for velocities and stresses in the symmetric form offers an essential advantage. Here, the elastic parameters are located only at the time derivatives, and we approximate not the derivatives of components of velocities and stresses, but these components in themselves. Such an approach essentially accelerates the convergence of our algorithm.

At the second step of the solution of our problem we employ the integral Laguerre transform with respect to the time (Konykh and Mikhailenko (1998), Mikhailenko (1999)). The integral Laguerre transform for a certain function $F\left(t\right)$ defines as:

$$F_j=\int_0^\infty F\left(t\right)\left(ht\right)^{-\frac{\alpha}{2}}l_j^\alpha\left(ht\right)d\left(ht\right) \tag{3}$$

with the inversion formula:

$$F(t) = (ht)^{\frac{\alpha}{2}} \sum_{j=0}^{\infty} \frac{j!}{(j+\alpha)!} F_j l_j^{\alpha}(ht), \tag{4}$$

where $l_j^{\alpha}(ht)$ are the orthogonal Laguerre functions.

The obtained after integral transformations system of equations is solved using the staggered finite difference schemes of fourth order of accuracy with respect to the coordinate *r*. The resulting system of algebraic equations is solved by iterative parallel methods, such as the conjugate gradient method. In this case an appropriate choice of the parameter *h* provides a rapid convergence of iterative methods.

Let us note that if elastic parameters of a medium depend only on the radial coordinate *r* in a certain part of the computation domain, then the resulting system degenerates, and we have the classical separation of variables in combination with a 1D finite difference technique (Alekseev and Mikhailenko (1977)).

Conclusion

We have presented the spectral finite difference algorithm for laterally heterogeneous earth models. This algorithm combines flexibility of the finite difference techniques for 1D problems with the accuracy of spectral representation. This approach allows varying computational costs and computer memory depending on the complexity of a medium on one or another part of a computational domain. In addition, if the elastic parameters of a medium are smooth functions of the azimuthal coordinate φ the method becomes much simpler and faster. This is particularly important for computation of seismic wave fields for 3D models of media.

References

1. *A.S. Alekseev, B.G. Mikhailenko,* Numerical modeling of seismic waves propagation in radial inhomogeneous Earth's model. *Dokl. Akad. Nauk SSSR*, 214 (1977), pp. 84 – 86, (in Russian).
2. *T. Furumuraa et al.,* Seismic wavefield calculation for laterally heterogeneous whole earth models using the pseudospectral method. *Geophys. J. Int.,* 135 (1998), pp. 845 – 860.
3. *G.V. Konyukh*, *B.G. Mikhailenko*, Application of integral Laguerre transformation in solving dynamic seismic problems. In: Proceedings of Institute of Computational Mathematics and Mathematical Geophysics. Mathematical Modeling in Geophysics, 5, pp. 79 – 90, Novosibirsk, 1998.
4. *B.G. Mikhailenko*, Synthetic seismograms for complex 3D geometries using an analytical-numerical algorithm. *Geophys. J. R. Astron. Soc.*, 79 (1984), pp. 963 – 986.
5. *B.G. Mikhailenko,* Spectral Laguerre method for the approximate solution of time dependent problems. *Appl. Math. Lett.,* (1999) 12, pp. 105 – 110.