A Study of a Gravity-Gradient Mixing Properties by the Means of a Direct Numerical Modeling.

Svetlana M. Garina, Marina Ye. Ladonkina[*], Nadezhda G. Proncheva,
Vladimir F. Tishkin, Nikolai V. Zmitrenko

Institute for Mathematical Modelling, Russian Academy of Sciences
4a, Miusskaya sq., Moscow, 125047, Russia

An analysis of the results of the gasdynamic calculations of a process of a turbulent mixing caused by development of Reyleing-Teylor instability is given at the present paper. Such the properties of the appearing turbulence as a mixing zone growth rate and a spectral dependence of kinetic energy upon a wave number were investigated. It was shown, that a direct modeling of the gravitational mixing process requires the sufficiently detailed meshes (cell number > 10^7) and, consequently, using of multiprocessor computers with a distributed memory.

Recently a considerable progress in the field of a computer technique and the numerical methods leads to a possibility of a direct mathematical modeling of the turbulent flows [1,2]. The gasdynamic code NUT [3], developed in IMM RAS, as well as another codes [4,5] quite adequately describe the experimental data. But, the peculiarities of an each numerical algorithm can bring the specific grid errors into a turbulence description, distort the flow pattern and do not reproduce all the peculiarities of a complex instability evolution. It means, that we need study in detail the sufficiency of the calculated data to the ideas about process under modeling. So, it is needed to use the sufficiently detailed grids in the simulations, which will be able to determine gravitational mixing characteristics, to study a fine flow structure and to investigate a spectral properties of turbulence using the difference methods. At the present work the direct modeling of a turbulent mixing is fulfilled for a problem of the Rayleigh-Teylor instability development. A contact boundary between two gases of a different density is subjected to hydrodynamics instability actions and in the course of time it forms a finite height zone. In this zone main part of a kinetic energy is concentrated and both the gases are mixed.

For this purpose in IMM RAS the program complex permitting to fulfill the calculations at the different parallel computers was elaborated. In the base of this system the code NUT was used. The NUT is a code to solve 3D gasdynamyc equations with the Eulerian variables. The main kernel of code is a scheme to solve the gasdynamic equations, which were taken in a form of the conservation laws of the mass, momentum and total energy. The discrete mesh is used, and at this work we use the fixed cubic spatial mesh. The space derivatives approximation is realized similarly to TVD schemes. For the fluxes determination through the bounds of the cells we use the Godunov's discontinuity disintegration scheme. The time derivatives approximation is realized by the explicit second order scheme (predictor-corrector).

The main theoretical ideas, concerning turbulence, are connected with Kolmogorov's spectral law [6], the timesquare law of a mixing zone width, the 'self-similar' density profile in a mixing zone.

It is important to establish what is a necessary mesh detail for a given difference method to adequately reproduce these characteristics of the turbulence. It is this case the total number of the cells found to be comparatively small, it means, that an effective study of turbulence is possible by means of a standard computer. In the opposite case, we need use the special multi-processor system with a distributed memory. At the present work we analyze the numerical results, obtained using the NUT code [3] with the meshes 100*100*200 (grid 1), 200*200*400

---

[*] Phone: +7(095) 9721791, fax: +7(095) 9720723, e-mail: ladm@imamod.ru

(grid 2) and 500*500*1000 (grid 3). For the second case (16 million of cells) the calculations can be carried out at multiprocessor system only (in our case MVS-1000).

The studied problem is posed by this way. The calculation region is a parallelepiped $0 \le x \le 1$, $0 \le y \le 1$, $-1.125 \le z \le 0.875$, A nonperturbed rest state is characterized by z = 0 as the z-coordinate of a contact boundary and the acceleration magnitude g =1. The acceleration is directed downwards, above the contact boundary ($0 \le z \le 0{,}875$) the heavy gas is placed ($\rho=3$), below ($-1{,}125 \le z \le 0$) – the light gas is placed ($\rho=1$). The pressure is distributed in accordance with a barometric formula, so $p(0)=5$. The used units for the physical values in this problem are conventional ones and they were obtained by using of the dimensionless procedure of the primary experimental data.

The flat contact boundary (z=0), was disturbed by the perturbation of the following form. At one layer of cells ($-0{,}005 \le z \le 0{,}005$, grid 1 or $-0{,}005 \le z \le 0$, grid 2) by a random way the density values $\rho = 1$ (light gas), or $\rho = 3$ (heavy gas). This choice was realized by the random number generator [8], which produces a random value ζ. This value is uniformly distributed at [0,1]. If the ζ >0.5 we have chosen $\rho = 3$, and $\rho = 1$ in the opposite case. Fig. 1 illustrates the dynamic of instability development.

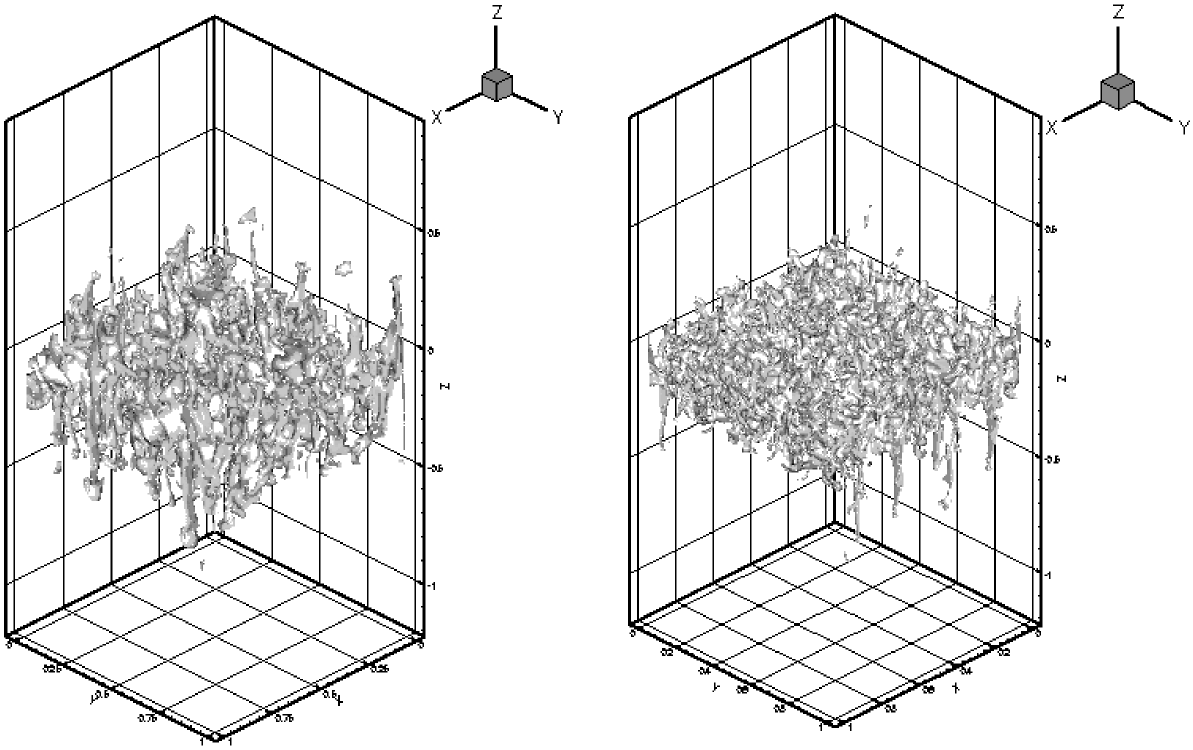

Fig.1. The isosurface *c=0.5* of volume concentration of the heavy gas in the simulation with grid 1(left) and 2(right) at *t*= 3.0.

One of the criterions of the turbulence development is an analysis of kinetic energy spectrum. For this purpose energy of the unit of a mass has to be analyzed, because the energy of the unit of a volume is proportional to the density, which is distributed in the mixing zone by a complex way. For a developed, isotropic and homogeneous turbulence the range of a wave number q exists, such, that the spectral energy amplitude $\tilde{E}(q) \sim q^{-5/3}$ [6] (so called the “inertial interval”).

The spectral analysis was fulfilled with a standard procedure [5-7]. The obtained dependencies are shown at Fig. 2.

It is well known that is wrong to consider the turbulence to be isotropic. To evaluate an anisotropy degree apart from $\widetilde{E}(q)$, it is useful to consider the 1D spectra. They are constructed by the rule $E^x(l) = \sum_m^{N_y} \sum_n^{N_z} (\widetilde{u}_{lmn}^2 + \widetilde{v}_{lmn}^2 + \widetilde{w}_{lmn}^2)$, and by the similar way for E^{Y}, E^{Z}, where $\widetilde{u}_{lmn}$, $\widetilde{v}_{lmn}$ and $\widetilde{w}_{lmn}$ are the spectral coefficients, $q = \sqrt{\frac{l^2}{L_x^2} + \frac{m^2}{L_y^2} + \frac{n^2}{L_z^2}}$.

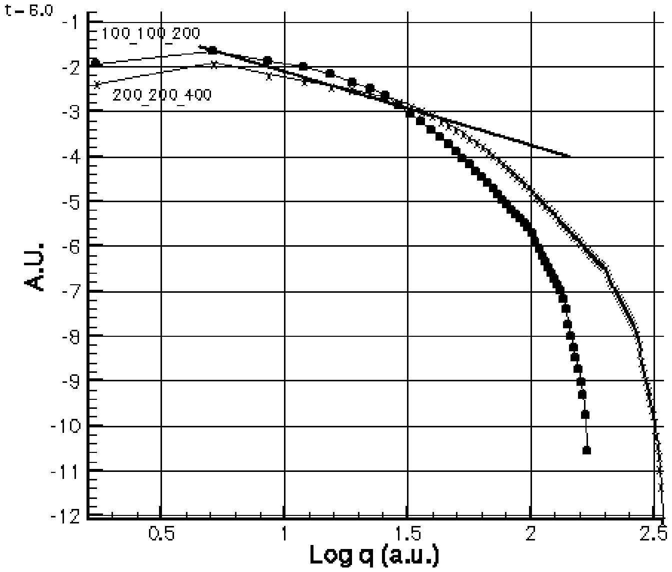

Fig. 2. Spectral properties of the turbulent mixing zone (TMZ) at the gravity-gradient mixing simulation. Circles indicate the data of grid 1, crosses indicate the data of grid 2, the direct line is inclined with an angle, corresponding to the exponent –5/3.

The general view of 1D spectra is found like 3D ones: a short range with $\widetilde{E} \sim q^{-5/3}$ is changed by a more extensive range with $\widetilde{E} \sim q^{-11/3}$.

The treatment results (see fig. 3) shows clearly the anizotropy: 1D spectra along X and Y directions approximately the same and differ from the spectrum along Z direction. Last direction is emphased, becouse the gravity force is directed along Z axis.

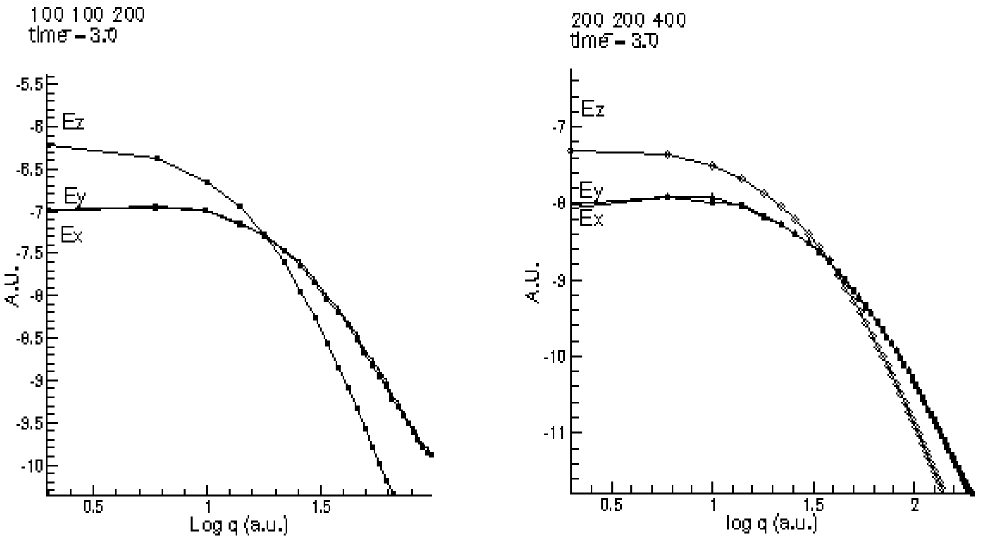

Fig. 3. 1D spectral characteristics of the TMZ at grids 1(left) and 2(right).

References

1. D.L. Youngs. Three-Dimensional Numerical Simulation of Turbulent Mixing by Rayleigh-Taylor Instability, Phys. Fluids A, 1991, v.3, N.5, pp.1312-1320.
2. O.G. Sin'kova, A.L. Stadnik, V.P. Statsenko, et al. Three-Dimensional Numerical Simulation of Gravitational Turbulent Mixing, Proceedings of Sixth International Workshop on the Physics of Compressible Turbulent Mixing, IUSTI, Marseille, 1997, pp.470-479.
3. V.F. Tishkin, V.V. Nikishin, I.V. Popov, A.P. Favorsky, The difference schemes of 3D gas dinamics for the problem on richtmyer-Meshkov instability development, (in Russian) Mathematical Modelling, 1995, vol.7, N.5, pp.15-25.
4. D.L. Youngs. Numerical Simulation of Mixing by Turbulent by Rayleigh-Taylor and Richtmyer-Meshkov Instabilities. Laser and Particle Beams, 1994, v.12, N.4, p.725-750.
5. N.N. Anuchina, V.I. Volkov, V.A. Gordeychuk et al. 3D Numerical Simulation of Rayleigh-Taylor Instability Using MAH-3 Code. Laser and Particle Beams, 2000, v.18, p.175-181.
6. A.S. Monin, A.M. Yaglom. Statistical Hidromechanics. (in Russian), Moscow, Nauka, 1967, 720 p.
7. L.D. Landau, Ye.M. Lifshits, Hydrodynamics. (in Russian), Moscow, Nauka, 1986, 736p.
8. Yu.L. Levitan, I.M. Sobol’. On a pseudo-random number generator for personal computers (in Russian), Mathematical Modelling, 1990, v.2, N.8, pp.119-126.