High Accuracy Simulation of Acoustic Noise from Sources in Pipes

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Introduction

Some industry fields, first of all, mechanical and aircraft engineering, require the solution of problems connected with acoustic noise propagation and generation in turbulent flows (mixing layers, jets, separated zones, external flows). That is why the development of algorithms for solving computational aeroacoustics (CAA) problems is very actual at present time.

There are known many efficient and widely used methods for solving aeroacoustics problems but there is no ideal one. Despite of the intensively growing power of different types of computer systems, the quality of numerical methods also needs in improving, in particular, by rising their accuracy. A high accuracy of methods is especially important in CAA where it's necessary to predict propagation of small scale acoustic disturbances.

One of the vital CAA problem is problem of modeling acoustic noise from sources in closed volumes. The paper describes some ways of solving this problem.

The paper comparatively reviews a family of high order methods of extended MacCormack schemes and their compact modification [cite{MC1},\cite{MC2}], new high order algorithm based on the synthesis of compact stencil space approximation and Runge-Kutta time integration procedure and the Dispersion Relation Preserving(DRP) scheme(\cite{DRP1}) in terms of acoustic sources modeling.

The paper represents the results of studying the set of high accuracy CAA methods as well as the outlook for their development and application for modeling acoustic sources in closed volumes with reflecting walls ("pipes").

All results describes in the paper calculated by **WHISPAR** CAA parallel program package developed in **IMM RAS**.

General problem formulation

The process of propagation of small (acoustic) disturbances is simulated by the linearized Euler equations representing a linear hyperbolic equation system either with the constant coefficients or with the coefficients depending on space coordinates.

Let us consider the Euler equations for inviscid gas flows:

$$\frac{\partial \mathbf{Q}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}+\frac{\partial \mathbf{G}}{\partial y}=0, \tag{1}$$

where

$$\mathbf{Q}=\begin{pmatrix}\rho\\ \rho u\\ \rho v\\ p\end{pmatrix},\quad \mathbf{F}=\begin{pmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ u(E+p)\end{pmatrix},\quad \mathbf{G}=\begin{pmatrix}\rho v\\ \rho uv\\ \rho u^2+p\\ v(E+p)\end{pmatrix}.$$

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Then let us decompose the vector of primitive gasdynamic variables as $\mathbf{U}=\mathbf{U}'+\overline{\mathbf{U}}$, $\mathbf{U}=(\rho,u,v,p)^T$, $\mathbf{U}'=(\rho',u',v',p')^T$, $\overline{\mathbf{U}}=(\overline{\rho},\overline{u},\overline{v},\overline{p})^T$, where the vector $\overline{\mathbf{U}}$ presents the mean flow values slowly varying (both on space and time) in relation to $\mathbf{U}$:

$$\frac{\partial\overline{f}}{\partial t}\ll\frac{\partial f'}{\partial t},\quad \frac{\partial\overline{f}}{\partial x}\ll\frac{\partial f'}{\partial t},\quad \frac{\partial\overline{f}}{\partial y}\ll\frac{\partial f'}{\partial t}.$$

And let us consider hereinafter that the flowfield $\overline{\mathbf{U}}$ is given and serves as a parameter of the equations (1).

If a matrix $\mathbf{P}$, $\mathbf{P}\mathbf{U}'=\mathbf{Q}'$ of transition from primitive to conservative variables is introduced and the terms of smallness order higher than the first in relation to $\mathbf{U}$ are neglected one can derive the following linear equation system

$$\frac{\partial\mathbf{Q}'}{\partial t}+\frac{\partial\mathbf{A}\mathbf{U}'}{\partial x}+\frac{\partial\mathbf{B}\mathbf{U}'}{\partial y}=0,\quad\text{or}\quad \frac{\partial\mathbf{Q}'}{\partial t}+\frac{\partial\mathbf{A}\mathbf{P}^{-1}\mathbf{Q}'}{\partial x}+\frac{\partial\mathbf{B}\mathbf{P}^{-1}\mathbf{Q}'}{\partial y}=0, \tag{2}$$

where

$$\mathbf{P}=\begin{pmatrix} 1 & 0 & 0 & 0\\ \overline{u} & \overline{\rho} & 0 & 0\\ \overline{v} & 0 & \overline{\rho} & 0\\ \dfrac{\overline{u}^2+\overline{v}^2}{2} & \overline{\rho u} & \overline{\rho v} & \dfrac{\gamma}{\gamma-1}\end{pmatrix},$$

$$\mathbf{A}=\begin{pmatrix} \overline{u} & \overline{\rho} & 0 & 0\\ \overline{u}^2 & 2\overline{u}\,\overline{\rho} & 0 & 1\\ \overline{u}\,\overline{v} & \overline{\rho v} & \overline{\rho u} & 0\\ \overline{u}\dfrac{(\overline{u}^2+\overline{v}^2)}{2} & \overline{\rho}\dfrac{(3\overline{u}^2+\overline{v}^2)}{2}+\dfrac{\gamma}{\gamma-1}\overline{p} & \overline{\rho u}\,\overline{v} & \dfrac{\gamma}{\gamma-1}\overline{u}\end{pmatrix},$$

$$\mathbf{B}=\begin{pmatrix} \overline{v} & 0 & \overline{\rho} & 0\\ \overline{u}\,\overline{v} & \overline{\rho v} & \overline{\rho u} & 0\\ \overline{v}^2 & 0 & 2\overline{u}\,\overline{\rho} & 1\\ \overline{v}\dfrac{(\overline{u}^2+\overline{v}^2)}{2} & \overline{\rho u}\,\overline{v} & \overline{\rho}\dfrac{(\overline{u}^2+3\overline{v}^2)}{2}+\dfrac{\gamma}{\gamma-1}\overline{p} & \dfrac{\gamma}{\gamma-1}\overline{v}\end{pmatrix}.$$

All the methods given below are applied to the system (2) of linearized equations taken in the conservative variables.

High accuracy numerical techniques

Let us describe a set of high accuracy algorithmic elements forming a basis of multi-block **WHISPAR** CAA program package.

The difference schemes constructed according to the **MacCormack** principle are the most dramatic examples of the 'predictor-corrector' schemes and based on the following algorithm: at the first stage a preliminary value of unknown function is calculated with the use of some difference operator ('predictor'), at the second stage this value is corrected ('corrector'), and at last a final value is calculated with the help of some procedure of averaging on 'predictor' and 'corrector' values (\cite{MC1}, \cite{MC2}).

The structure of space derivative operators D_x and D_y is, the above given scheme has an approximation order of $O(\tau^2)$ on time, and total approximation order of $O(\tau^2+h^m)$ (hereinafter it is noted as (2,m)), where m is an approximation order of space derivatives by the corresponding operators.

Using MacCormack schemes is one of the simple way of solving the 2D hyperbolic equations system like (1).

Another class of finite difference schemes used for solving problem of acoustic source modeling in this paper are difference schemes which have the same dispersion relations as the original partial differential equations and referred as **dispersion-relation-preserving (DRP)** schemes \cite{DRP1}. Base DRP algorithm can be used for approximation of space and time derivatives.

In this paper MacCormack (2,4) scheme (based on approximation of space derivatives on the 5-point stencil), MacCormack scheme based on compact stencils, compact modification of Runge-Kutta scheme, Jameson integration based methods \cite{J1} and classic DRP scheme have been used.

Boundary conditions

Solving acoustic noise propagation in pipes problem require good nonreflecting boundary conditions for modelling inflow and outflow areas. In this work for modeling inflow and outflow areas **Tam boundary conditions** and **perfect match layer (PML)** algorithms have been used.

Tam boundary conditions are one of the most widely used nonreflecting boundary conditions based on the far-field asymptotic solutions and proposed by Tam (*Christopher K.W. Tam*, \cite{B1}). Radiation boundary conditions at the inflow boundary (of 2-dimensional domain) and boundary conditions at the outflow boundary are very useful for solving problem described above.

Another type of widely used boundary conditions for numerical simulation with an open domain is the buffer zone techniques. In these techniques we extend a computational domain by adding an extra zone where the numerical solution is damped in different ways. Perfect match layer **(PML)** can be considered as a special type of buffer zone conditions. According to this technique, special PML equations are solved in the buffer zone, in order to damp the outgoing waves supported by the Euler equations \cite{PML}.

The PML equations provide the absorption of waves without the reflection. In the presence of a mean flow there exist acoustic waves which in traditional PML formulation can be amplified. *Fang Q. Hu* \cite{PML} proposes a new stable PML formulation, to overcome these difficulties. In the same manner as in the traditional PML formulation, the stable unsplit PML equations are solved in the buffer zone surrounding the domain.

Numerical results

Benchmarks description. Solving of central problem consist of three stages. Firstly we need good acoustic source model capable of accurate modeling of acoustic disturbances propagation. In the second place good nonreflecting boundary conditions are needed. Final stage is solving complete acoustic source in pipe modeling problem is testing quality of integration selected numerical methods and boundary conditions. That is why numerical part of this investigation consists of solving three special tests.

In first test problem (*TEST* 1) acoustic source was placed in the center of square area with free (nonreflecting) walls. The goal of first test was testing quality of selected numerical methods and testing quality of nonreflecting boundary conditions.

The second test problem (*TEST* 2) was taken from the Proceedings of *ICASE/LaRC Workshop on Benchmark Problems in Computational Aeroacoustics* (*CAA*) \cite{T1} (Problem 1, Category 4). The main goal of this test was verification quality of reflecting and nonreflecting boundary conditions by comparing numerical and analytical solution.

And the last test problem (*TEST* 3) was complete "source-in-pipe" test.

In all test problems the mean field following (2) terms was formulated as follows:

$$\bar{\mathbf{U}} = \begin{pmatrix} \rho \\ u \\ v \\ p \end{pmatrix} = \begin{pmatrix} 1.0 \\ M \\ 0 \\ 1/\gamma \end{pmatrix},$$

where M is value of horizontal mean flow velocity.

All test problems described above have been solved with different values of mean flow velocity (M range from 0.1 up to 0.8).

Test results. The test problems are solved on a uniform mesh of size 200*200 and Courant number 0.1 for DRP scheme and 0.3 for MacCormack schemes and compact Runge-Kutta scheme.

As you can see developed methods are capable of solving linear CAA problems with good precision and realized boundary condition (nonreflecting(see fig. 1) and reflecting (see fig. 2) works good with different mach numbers of mean flow in solving problem of acoustic source modeling in pipe.

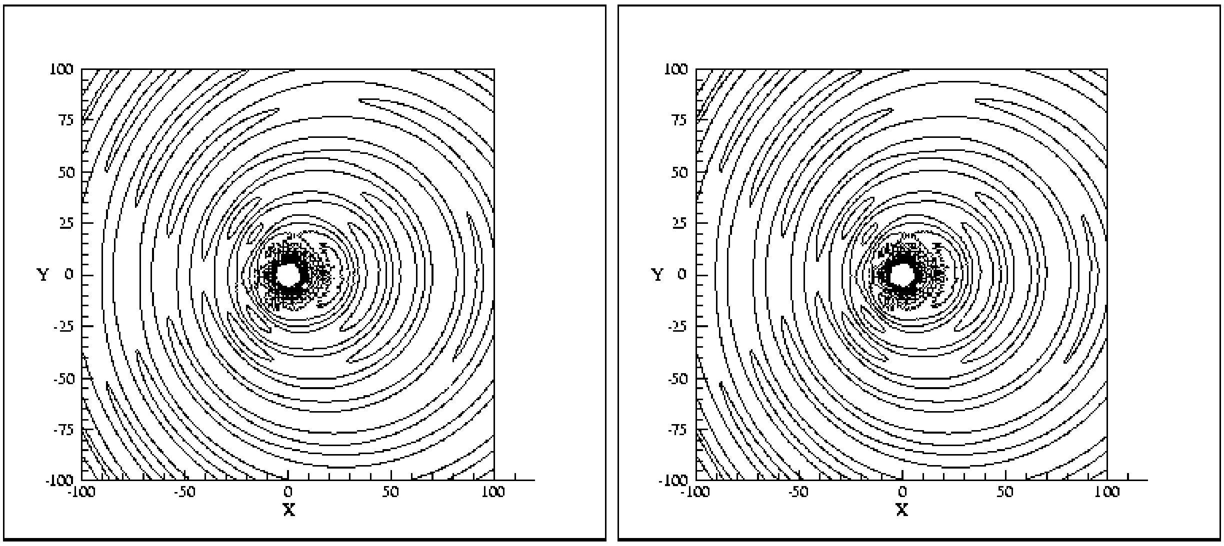

Fig. 1. Density field for *TEST* 1 (mean flow (M=0.3), time 300(*left*), time 500(*right*)).

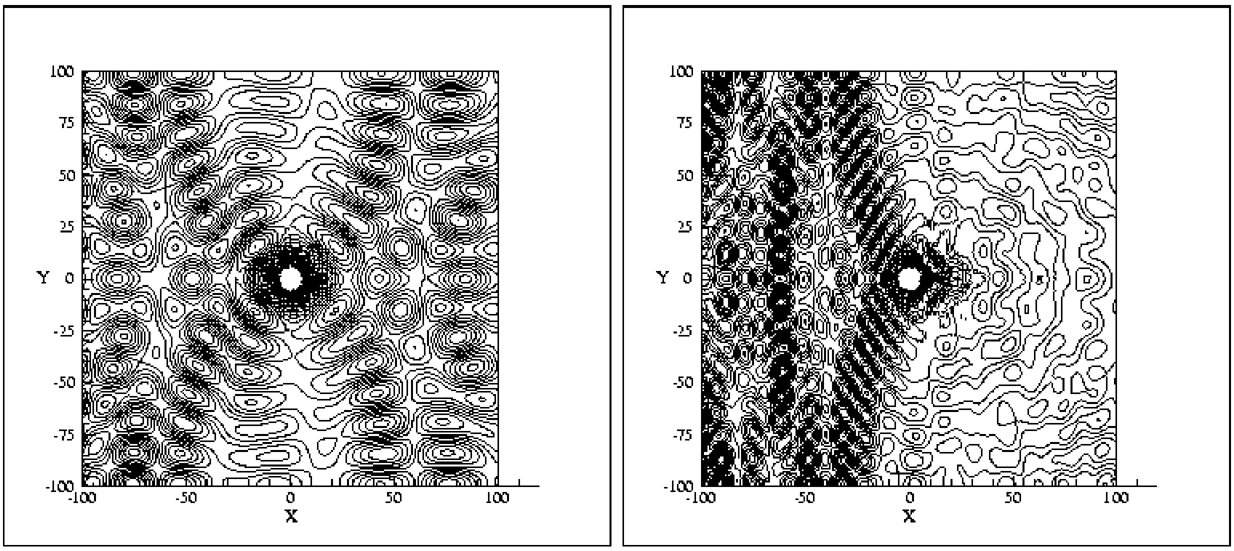

Fig. 2. Density field for *TEST* 3 (time 3000, mean flow M=0.1(*left*), M=0.6(*right*)).

Conclusion

The paper presents way of solving one of important engineering problem problem of modelling acoustic source in closed volumes.

Using parallel program package **WHISPAR** allows perform wide range of numerical tests with different conditions in purpose of constructing most effective combination of numerical method and boundary conditions for accurate modeling of acoustic source in pipe.

The testing results confirm a high accuracy of implemented algorithms. Both compact Runge-Kutta and DRP schemes perfectly agree with the exact solutions of benchmark problems. These schemes are well combined with the Tam and PML non-reflecting boundary conditions and provide an accurate solution.

The above remarks provide a choice and easy construction of complete numerical method for applied aeroacoustics problems and research purposes. The algorithmic capacities of the package are supposed to grow. It is to be completed both by model extensions (firstly, inclusion of viscosity for linearized Navier-Stokes equations and account for nonlinear effects) and using axially symmetric and full 3D base models.

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