Parallel Adaptive Solution of Stokes and Oseen Problems on Unstructured 3D Meshes

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Robust parallel algorithms for adaptive solution of boundary value problems for PDEs have been of interest to engineers and mathematicians over many years. Significant improvement of the accuracy of approximation through the adaptive distribution of mesh elements rather than an increase of their number enables to solve large problems arising in applications. Parallel techniques for generation of adaptive meshes and solution of the associated discrete problems extend computational capabilities considerably.

A novel technique of the parallel adaptive solution of Stokes and Oseen problems is presented. Its constituents are parallel adaptive mesh generation and parallel solution of the discretized problem. Both stages are independent of each other and are constructed to be as much independent of the underlying problem as possible.

A lot of popular adaptation algorithms are based on a posteriori error estimators. Besides the discrete solution, the error estimators require additional data of the problem which reduces their flexibility. An alternative approach is based on a black-box Hessian recovery technique. The objective of this technique is to construct a mesh which is quasi-uniform in a metric field generated by the discrete Hessian recovered from the discrete solution. If the solution features an anisotropic behavior, the adaptive mesh turns out to be anisotropic in order to fit very well the solution variations. Nowadays, this is the most promising problem independent approach for adaptive mesh generation. The parallel black-box 3D mesh generator based on the Hessian recovery technique is the first stage of the method.

Any adaptive technique has to be equipped with a solution procedure. To our best knowledge, there is a lack of efficient parallel black-box solvers even for elliptic problems. The majority of available algorithms are problem dependent, i.e., they have to be tuned up for the particular problem. We propose a new parallel black-box solver independent (as much as possible) of the underlying elliptic problem. The solver is based on the domain decomposition (DD) technique and an efficient sequential subdomain black-box preconditioner which is assumed to be at hand. In our work, we take advantage of the algebraic multigrid (AMG) preconditioner known as AMG1R5 (J.Ruge,K.Stuben, Release 1.5, 1990). In our DD framework a block-diagonal preconditioner based on the AMG solver in subdomains is corrected by a special smoother. The smoother enables us to formulate an iterative solver with the convergence rate independent of the number of processors (subdomains), problem coefficients, and only slightly sensitive to the problem size. The overall preconditioner is very simple to implement and parallelize.

The Stokes and Oseen problems are elliptic only in the subspace of divergence-free functions. The iterative solution of the discrete problems does not need to stay in the divergence-free subspace. We consider parallel block-diagonal preconditioners for the saddle-point matrices of MFEM approximations for Stokes and Oseen problems. The basic ingredient of the preconditioners is the discussed above black-box parallel DD preconditioner for an elliptic operator. A test case (see Fig.) with point and edge singularities and a boundary layer is considered. Parallel and numerical properties of the solvers are considered.

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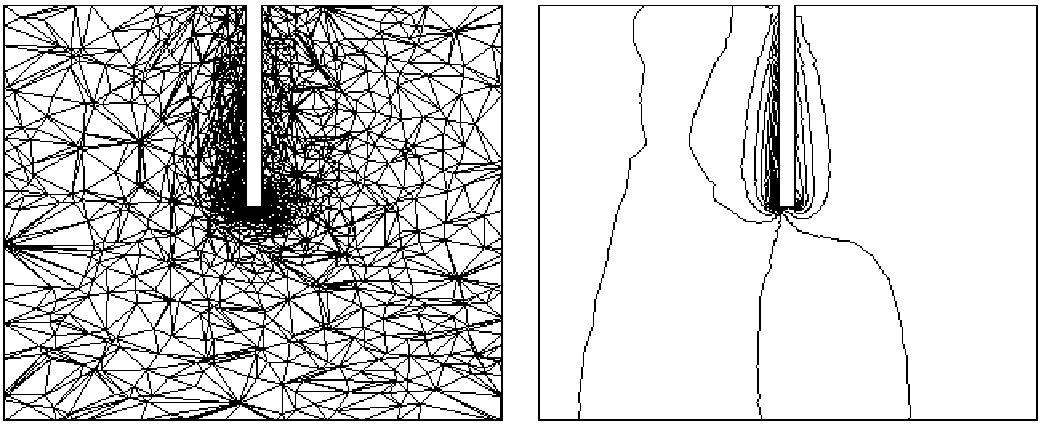

Fig. 1. An example of the adaptive mesh and the pressure contours for the test case. Cross-section of the 3D mesh and the field.

References

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