On Arbitrary-order Schemes for Parallel CFD Calculations

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To increase approximation orders of traditional numerical methods for PDE's admitted by exact solutions smoothness, one usually tries to increase numbers of basis functions underlying their discretizations (by increasing, for example, numbers of degrees of freedom defining orders of local polynomial interpolants). In many cases, it may complicate the resulting formulas and create some problems when realizing the constructed algorithms. An alternative way was suggested in [1]. Its essence is using linear combinations ("multioperators") of basis operators, each basis operator being formed by specifying a value of a parameter in one-parametric families of compact approximations. Fixing M values of the parameter, it is possible to construct multioperators with approximation orders which are linear functions of M, that is, in fact, formally with arbitrary orders. An important property of multioperators is the possibility to calculate their actions on known grid functions in a parallel manner using M processors, each one being involved in calculations with basis operators. Thus no extra computational costs are needed when increasing approximation orders by increasing M.

The present talk concerns with further development of the multioperators-based schemes for parallel CFD applications. To construct basis operators and corresponding multioperators for convection terms, one-parametric families of third- and fifth-order accurate Compact Upwind Differencing (CUD) formulas from [2] were used. The resulting schemes of (M+2)-th and (M+4)-th order respectively are described. They are capable to provide high resolution of fine scales and at the same time they contain a built-in filter of high-frequency numerical noise. Numerical examples relevant to vorticity and vortex wakes evolutions are presented showing speed-up measured by orders of magnitudes.

As an extension of the multioperators principle, a way of constructing 2M-th order centered approximations to the fluid dynamics equations and finite-volume multioperators schemes are presented. The main idea here is using basis operators which are rational functions of three-point centered operators with artificially introduced parameters. Centered multioperators approximations are more accurate than their upwind counterparts but they are more appropriate for smooth solutions without steep gradients. The potential for extremely high accuracy of the technique is shown by presenting results of several benchmark calculations.

Finally, the domain decomposition strategy of combining the multioperators method with a specially designed meshless technique based on using Radial Basis Functions in a "finite difference mode" in the case of complicated geometries is outlined.

References

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