Influence Functions Method and Parallel Calculations for Numerical Solution to Boundary-Value Problem of Radiation Transfer Theory In Two-Media Systems

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Introduction

Information and mathematical techniques as well as software tools of computer applications are initial to realize the proposed research designed for mathematical modeling of natural processes and calculation procedures on the parallel computer systems. These techniques and applications are linked with the environmental and technological safety, industrial pollutions, natural disasters and anthropogenic influences, emergency situations.

Additionally to the common-used hypothesis of global warming due to as assumed by the "green-house gases" effect, the problem of studying the optical and meteorological as well as climatic behavior of the complicated "atmosphere-ocean-land-ice-biosphere" system with many its feedbacks is seen to be urgent. Thorough analysis of data sets obtained from global observing systems is important for diagnostic and predictability purposes. These data sets are typically analyzed along with modeling results to use remote sensing and monitoring techniques while employing the related scenarios of emergency situations and natural disasters evolution in the atmosphere and on the Earth surface. The relevant understanding, regular monitoring and predictability of weather and climate phenomena and their extreme manifestation under natural and anthropogenic influences are of great importance to sustainable development and assessment of consequences of evolution of the Earth as a planet and to investigate the life-supporting systems for the human community.

All the listed factors open up an opportunity to evolve the subsequent models of an interaction between radiation and natural media based on particular approximations of the optical transfer operator (OTO) as a comprehensive mathematical tool for the simulation techniques. These models are employed in multi-dimensional problems of radiation correction of the ocean from space, in theories of light and image propagation in turbid media, and in theoretical and computational fundamentals of optical-electronic remote sensing systems. New results are presented here given by sets of basic models to compute the optical radiation transfer in the atmosphere-ocean system (SAO). The basic sets of the influence functions (IFs) of the atmosphere and ocean are determined. These sets of the IFs are invariant with respect to the reflection and refraction properties of the air-water boundary. The newly defined techniques are proposed to compute the solar radiation field in the atmosphere-ocean system as separated solutions for the atmosphere and the ocean.

Background

Multidimensional spherical and plane-parallel models of the radiation transfer in the atmosphere - surface system (SAS) are under way together with the relevant computer

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applications. These models are developed to find numerical solutions of the boundary value-problem (BVP) for the stationary equation of the monochromatic or quasi-monochromatic radiation transfer in the atmosphere with scattering, absorbing, emitting and refracting properties. The spatial structure of the atmospheric media (both in spherical and plane-parallel approaches) is underlied by any inhomogeneous reflecting surface (land, ocean, etc.) with the specified boundary conditions for cloud or hydrometeor layers.

If to consider the atmosphere as a channel or a unit of an optical system in the radiation transfer theory, the optical transfer operator (OTO) can be defined in a linear approach assuming the applicability of the superposition law for the integral expression in the equation of transfer. This approach is called by us as the method of the influence functions (IF) [1-6].

Linear-system approximation with models for the optical transfer function and the point-spread function formulated in physical terms is commonly employed in multidimensional problems of radiation correction for remote sensing of objects and the natural environment, in processing of optical information, in theories of sight and image propagation in turbid media, and in theoretical and computational fundamentals of optical-electronic remote-sensing systems. Problems of radiation propagation in three-dimensional plane layers with horizontally non-uniform reflecting boundaries are more complicated, because certain theoretical principles implicit in the theory of linear systems, such as the invariance principle, optical reciprocity theorem, and invariance with respect to planar translations, are not valid.

The constructed base of a variety of mathematical models using the vector IF and the vector OTO values can allow to apply the proposed new algorithms of numerical modeling of the polarized radiation transfer in the systems "atmosphere - land", "atmosphere - ocean", "atmosphere - cloud", "atmosphere - hydrometeors", "atmosphere - vegetative community", “atmosphere - smoke”. Besides that, the radiation correction procedures are outlined in the related methods of remote sensing applications. Additional applications are concerned the theory of vision and the transfer theory of images through any opaque polarizing media.

The radiation field in the SAO is calculated using the optical transfer operator characterized by the influence functions of the atmosphere (IFA) and ocean (IFO) in our models. This new original approach is formulated in the context of a classic linear-system approach. The problems set up in these models to determine the IFA and IFO are identical to a routine one-dimensional problem of the radiation transfer theory in a planar layer illuminated by an external parallel flux of incoming radiation. The proposed techniques to solve the formulated problems depend on the optical and physical characteristics of natural media. The atmosphere can be cloudy or cloudless. The ocean is considered as a finite or semi-infinite layer.

The ocean is a rather conservative medium as compared to much more changeable optical and meteorological parameters of the atmosphere. The proposed methods of IFA and IFO are effective just for such situations of different optical properties of these natural media. In the final run, the relevant research is dedicated to finding the sunlight distribution in the oceanic waters by using remote sensing data for the bioproductivity assessment.

Mathematical statement of the problem

Let us consider a plane layer of natural media, horizontally unbounded $(-\infty < x, y < \infty)$ and of finite height $(0 \le z \le H)$, $r_\perp = (x, y)$. The set of all directions $s = (\mu, \varphi)$, $\mu = \cos\vartheta \in [-1,1]$, and $\varphi \in [0, 2\pi]$ is a unit sphere $\Omega = \Omega^+ \cup \Omega^-$, where Ω^+, Ω^- are the hemispheres of propagation directions for descending, transmitted radiation $(\mu \in [0,1])$ and ascending, reflected radiation $(\mu \in [-1,0])$, respectively. For the needed convenience of representation of the boundary conditions in the formulated problem, let us define the following sets

$$t = \{(z, r_\perp, s):\ z = 0,\ s \in \Omega^+\},\ b = \{(z, r_\perp, s):\ z = H,\ s \in \Omega^-\}.$$

The first boundary-value problem for the three-dimensional transfer equation is formulated as that referred to the non-reflecting boundary with linear operators:

$$K\Phi = 0, \qquad \Phi\big|_t = 0, \qquad \Phi\big|_b = E(r_\perp, s); \tag{1}$$

$$K \equiv D - S; \quad D \equiv (s, grad) + \sigma_{tot}(z); \qquad S\Phi \equiv \sigma_{scat}(z)\int_\Omega \gamma(z,s,s')\,\Phi(z,r_\perp,s')\,ds'.$$

We can obtain a solution of the stated problem (1) in the form of a linear functional

$$(\Theta, E)(s^-; z, r_\perp, s) \equiv \frac{1}{2\pi}\int_{\Omega^-} ds'' \int_{-\infty}^{\infty} E(s^-; H, r'_\perp, s'')\,\Theta(s''; z, r_\perp - r'_\perp, s)\,dr'_\perp,$$

where the kernel is the above-mentioned IF represented as $\Theta(s^-; z, r_\perp, s)$ which enables to solve the asserted boundary-value problem in the form

$$K\Theta = 0, \qquad \Theta\big|_t = 0, \qquad \Theta\big|_b = f_\delta(s^-; r_\perp, s); \qquad f_\delta(s^-; r_\perp, s) = \delta(r_\perp)\delta(s - s^-).$$

Results

The general boundary-value problem can be written in the following way

$$K\Phi = 0, \qquad \Phi\big|_t = 0, \qquad \Phi\big|_b = \varepsilon R\Phi + \varepsilon E(r_\perp, s). \tag{2}$$

The well-known representation of the gained solutions (2) can be ascertained in the form of the following series relative to the multiplicity factor of the reflections on the lower level of the underlying surface:

$$\Phi = (\Theta, \mathrm{V}E), \qquad \mathrm{V}E \equiv \sum_{n=0}^{\infty} G^n E, \qquad Gf \equiv R(\Theta, f).$$

The asymptotically exact solution of the boundary-value problem in the transfer theory can be written as

$$K\Phi = 0, \qquad \Phi\big|_t = 0, \qquad \Phi\big|_b = 0, \tag{3}$$

$$\Phi\big|_{d1} = \varepsilon(R_1\Phi + T_{21}\Phi + E_1), \qquad \Phi\big|_{d2} = \varepsilon(R_2\Phi + T_{12}\Phi + E_2)$$

where the border between the media (the atmosphere and the ocean) is divided in such a way that the predetermined illumination conditions are given by $\mathbf{E} = \{E_1, E_2\}$. Here new sets are introduced

$$d2 = \{\,(z, r_\perp, s):\; z = h,\; s \in \Omega^+\}\;; \qquad d1 = \{\,(z, r_\perp, s):\; z = h,\; s \in \Omega^-\}\;.$$

Any single event of the interaction between the radiation and thus divided boundaries is described by the operators of reflection R_1, R_2 and transmission T_{12}, T_{21}, where the indices 1 and 2 are to be related to the layers with $z \in [0, h]$ and $z \in [h, H]$, respectively.

To solve the problem (3), we introduce the perturbation theory series

$$\mathbf{\Phi}(z, r_\perp, s) = \sum_{n=1}^{\infty} \varepsilon^n \mathbf{\Phi}_n(z, r_\perp, s)$$

with the above parameter $0 < \varepsilon \le 1$.

Using the coming up vector of the influence functions $\mathbf{\Theta} = \{\Theta_1, \Theta_2\}$, we can introduce the vectorial operation describing any single event of the interaction between the radiation and the divided boundary in accordance with the aforementioned procedure with multiple scattering in either media $(\mathbf{f} = \{f_1, f_2\})$ taking into account:

$$[\Pi\mathbf{f}](h, r_\perp, s) \equiv P(\mathbf{\Theta}, \mathbf{f}) = \begin{bmatrix} R_1(\Theta_1, f_1) + T_{21}(\Theta_2, f_2) \\ T_{12}(\Theta_1, f_1) + R_2(\Theta_2, f_2) \end{bmatrix}, \qquad P = \begin{bmatrix} R_1 & T_{21} \\ T_{12} & R_2 \end{bmatrix},$$

where $(\mathbf{\Theta},\mathbf{f}) = \{ \ (\Theta_1, f_1),(\Theta_2, f_2) \ \}$ is a vectorial functional. One can prove that two successive n-approximations are connected by the recurrent relations

$$\mathbf{\Phi}_n = \left(\mathbf{\Theta}, P\mathbf{\Phi}_{n-1}\right) = \left(\mathbf{\Theta}, \Pi^{n-1}\mathbf{E}\right).$$

As a result of the proposed improvements, we can obtain the asymptotically exact solution as

$$\mathbf{\Phi} = \sum_{n=1}^{\infty} \mathbf{\Phi}_n = \left(\mathbf{\Theta}, Z\mathbf{E}\right); \qquad Z\mathbf{E} \equiv \sum_{n=0}^{\infty} \Pi^n \mathbf{E}$$

is the sum of a von Neumann series with respect to the multiplicity of radiation proceeding through the divided boundary of the media including the contribution of multiple scattering in each media.

Conclusion

The influence functions method has been shown to be one of the most effective techniques of solution of the boundary value problems of the radiation transfer. The presented constructive approach is effective for mathematical modeling of radiation transfer in natural media to solve the multidimensional problems using multiprocessor computers with parallel structure.

Three main conclusions can be extracted from many previous results concerning the state-of-the-art of the problem of measurement data registration and remote sensing. The first is related to finding implicit ways of the registration of any refraction surface from spacecrafts. The second is concerned the opportunity to make these procedures by the explicit way of calculating through the application of the IF method. The third is given by the method of value function and adjoint equations. The IF term enables to integrate any type of singularity and diffusive characteristics of the relevant sources and boundaries.

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