Viscous Gas Flow Simulation on Nested Grids Using Multiprocessor Computer Systems

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The problem of high accuracy resolution of the solution particularities in small regions often occurs during numerical modeling a large amount of modern problems of mathematical physics. These peculiarities may be the result of as physical processes as problem geometry (for example, detonation or combustion processes, solitons motion etc.). Using of regular non-uniform meshes in such situation leads to inevitable spreading of fine mesh into regions where it is not necessary. Fig. 1 demonstrates an example of such mesh for simulation of viscous gas flow around a solid body. We do not need the fine grid along lines *BE* and *BD*. So we will have a waste of computational time, especially in 3D case. It is naturally in this situation to use multiblock grids where different subregions have their own grids. A variant of such approach is the use of nested (or locally refining) grids, which are under consideration in this paper.

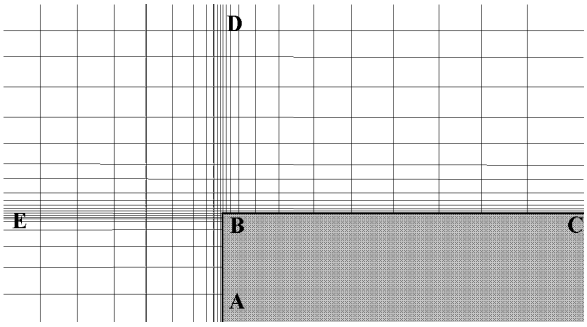

Fig. 1.

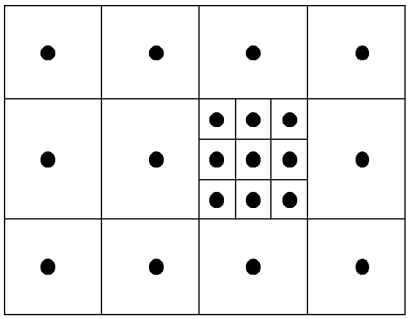

Fig. 2.

The use of nested grids for the local peculiarity resolution has several decades' history and now it is a gustily developing branch of numerical mathematics. In doing so main effort are concentrated on *finite elements methods* [1-2]. The presented paper is devoted to developing and parallel realization of explicit *finite difference schemes* for the numerical solution of gas dynamic problems on such type of grids. The main idea is to divide one or more cells of regular (maybe rough) grid into some amount of small cells (see Fig. 2). If these new cells are not sufficiently small we may repeat the refining procedure and divide some of them into more small cells and so on.

The schemes are under consideration, which are constructed as an approximation of conservation lows. So, the first problem arising in our investigations is approximation of fluxes between cells of different sizes. Two methods of discrete solution prolongation were tested on the Dirichlet problem for 2D heat conduction equation: by piecewise constant and piecewise linear functions in the "large" grid cell (Fig. 3).

The numerical experiments on the test problems with the regular and singular solutions show that the piecewise constant prolongation is preferable especially when solution has singularity. In this case accuracy was 3 times higher for the piecewise constant prolongation then

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for piecewise linear one. Naturally the use of nested grid does not advantage when the solution is regular.

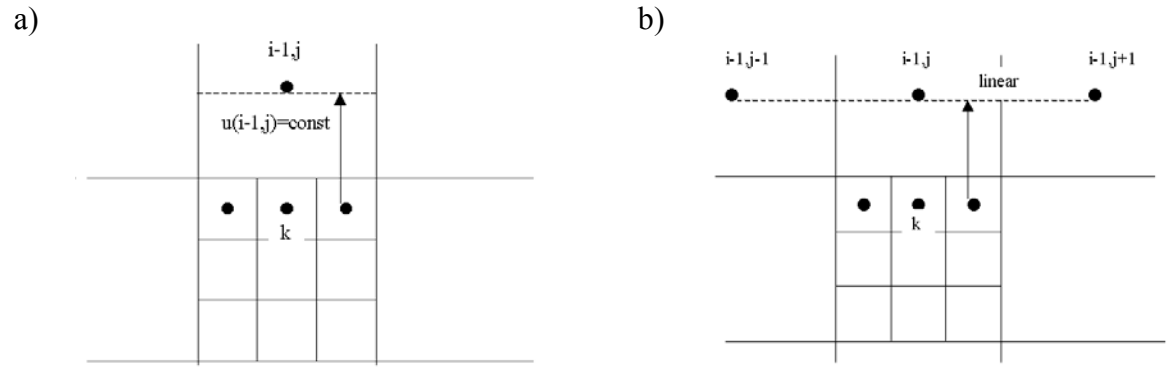

Fig. 3. Piecewise constant (a) and piecewise linear (b) prolongation of discrete solution.

These results were applied to the gas dynamic flow simulation. The parallel program complex was developed for numerical modeling of viscous 2D gas flow on rectangular nested grids. The kinetically consistent finite difference (KCFD) schemes [3] were taken as the basis for our calculations. The successful experience in solving various gas dynamic problems by means of such schemes showed that they describe viscous heat conducting flows as good as Navier-Stokes equations where the last are applicable. In addition to this these schemes permit to calculate oscillating regimes in super- and transonic gas flows, which are very difficult for modeling by means of other algorithms. KCFD schemes can be easily adapted to parallel computers with MIMD architecture. These schemes are homogeneous schemes i.e. the one type of algorithm describe as viscous as inviscous parts of the flow. We used the explicit schemes, which have soft stability condition. The geometrical parallelism principle have been implemented for constructing their parallel realization. This means that each processor provides calculation in its own subdomain. The explicit form of schemes allows to minimize the exchange of information between processors. Having equal number of nodes in each subdomain the homogeneity of algorithm automatically provides load balance of processors. So, the next problem was the problem of nested grid partitioning.

Naturally, nested grids may be considered as unstructured mesh and we can use one of the algorithms for unstructured mesh partitioning. But this approach wouldn't permit us to use the fact that the original mesh is regular. So we decided to exploit our experience in rectangular grid partitioning [4]. Each cell of original mesh was ascribed a weight 1. If in a process of refining a cell was divided into N small cells, it was ascribed a weight N. After it the computational region is divided into needed number of subdomains with approximately equal total weights. If the mesh refinement is too deep, some of original cells weights may be too large for one subdomain. In this case we may at first treat each of such cells separately using for them the same algorithm. After allotting a needed number of small cells to a subdomain we subtract this number from the weight of treated large cell. Such multistep partitioning algorithm was realised in our program complex.

The program complex was tested on the problem of supersonic viscous gas flow over a flat plate. The original rectangular grid was refined near the plate tip and in the boundary layer. The calculations were held for the total number of cell over 50000 on multiprocessor computer system MCS-1000. The parallelization efficiency for up to 20 processors was more than 90%. In the nearest future it is supposed to expand this algorithm for the simulation of 3D viscous gas flows.

References

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