Method of Virtual Z - cells via of Grids Superposition

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About increase of a solution exactitude of transverse propagation on a sequence of a grids superposition with a distinguishing virtual cells form.

Key words: transverse problem of gasdynamic, grids superposition, virtual Z-shape cells.

Introduction or approach to a problem

It is well known [1, 2], that in a multidimensional advection algorithms greatest errors arises if a medium have transverse motion in a direction to the one corner of a cell enclosing a settlement node of a grid. This is well known transverse propagation problem [3]. The usual process of a mesh refinement does not decide this problem in principle. However it can be radically decided, if alongside with a primary cell (basic idea) to enter into account else of other cells constructed on the same grid nodes (grids superposition, GSPM).

To a visual illustration of basic idea there is used at first the fragment of a regular grid forming the elementary cells - quadrates, as shown in fig.1a. It is clear for us, the nodes set forming a grid, basically can be selected completely arbitrary. But the same arbitrariness is allowable to a choice of topology 2D (3D) of cells, that was reflected in practice as emerging of algorithms realizing finite difference methods with three, four, five, six and unregular mixing sides of (2D) or (3D) – cells.

The basic necessity in cells has arisen with using to construction of discretized form of governing equations by the integration method over the control volume, which in themselves do not put forward to their form of certain special requests. Without loss of generality but for a determinancy here and now also we shall accept, that the speech will goes about use <u>all</u> of the well-known schemes on three-dot templates into some dimensional direction.

The grids features

Well, let's now to fix position of primary grid nodes and (basic action) then we shall redraw all work-space by diagonal lines. As the result we have duplicated a work-space, which contain the position of primary nodes of a grid and to differ only by new and opposite declination of diagonal bands. From all there fix some two, which are shown on fig.1b. As usually we can use nodes of a primary grid for a partition of a space in the each band on number of cells. Thus along diagonals of primary cells we shall received a number of new cells of the rectangular form, which twice densely covering same work-space. Now is compatible all cells of all bands and for any one node we shall consider them as though rodding them on one "needle".

We can use these new cells and same the three-dot template for account of the same parameters, which determined in same primary nodes of the same grid but already in new dimensional direction: along of diagonals. Finally there is nontrivial fact (see fig.1b): the forms of cells for the crossed diagonals can not coincide in space, though are identical in our last case. To my mind it is all necessary to know of the novelty to use of the grids superposition method.

Advantages and benefits or what play?

First of all we received the extension of the common template (for 2D grid - from five to nine-nodes; for 3D - from seven to 27) for account of parameters in each node of a primary grid. Second, the solution in a one node is to yield with using same algorithms for each direction, but

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already with allowance for new turn (rotated) position without using algorithms of a rotation. Third, now to determination of diagonal streams (that especially is important to realization of high resolution methods and to solving general transverse advection problem) there is no need to calculate matrixes of the mixed derivatives for it (as in [3-5]).

Fourthly, the orthogonality of coordinates of directions in our special case of a primary grid forming regular cells: it is saved for primary guadrates, so for diagonal bands too. The significance of magnitude of a volume of cells is same, it's constant. A case of primary rectangles the orthogonality is absent but there is constant a significance of a volume of cells, connected to the settlement node, despite of a modification of their forms and orientations of directions. The constancy of volume of the enclosed cells forming any four-coal cells can be one more criterion alongside with other [6].

Fifthly, as a result there coincide all specific parameters, which is associated usually with a node of grid. Finally, one the same data set is used to increase of an exactitude of a numerical solution in contrast from represented in [4–6].

The some features of GSPM algorithm

There are present not less than two possibilities depending on features of templates and algorithms. Further it's possible to act as an example so:

- to execute calculations twice: on usual set of cells to obtained values $(Ui)_1$ in nodes and on new diagonals set for ($(Ui)_{2,3}$); storing outcomes and then them somehow "weighing"of values;
- to execute the same calculations, if there is primary using a unsplit pointwise algorithm.

For "weighing" let's used of some parameter ak as the "weight", connected with a set of cells; k = 1, 2 (3). Then there is necessary to find some function for these "weighing" parameter. For example the a1 can be defined as follows: a1 = cos2(2α), where α - angle of a vector of the velocity as an axample. In conclusion we shall define final significance anyone of magnitudes in a node as an example so: $U_i^{n+1} = a_1 \cdot (U_i^{n+1})_1 + (1 - a_1) \cdot (U_i^{n+1})_2$.

The error of movement will be least and appropriate realized for this set of cells of templates and schemes if there is the overlapping of a velocity with one from chosen directions (including diagonal bands) in nodes. If before the transverse motion along a diagonal of primary cells had the greatest error, now here $a_1 = 0$ and the error decreases up to a minimum, as now coincides movement on normal to an edge of a new cell and is determined by a solution of $\left(U_i^{n+1}\right)_2$. Obviously, generally magnitudes a_k are different from 0 and 1. Basically, it is free parameters, which can be selected optimum for a sequence of the grids from conditions of increase of an exactitude of a solution of a concrete problem.

In the general case, one can select now the optimum shapes of cells, which have depending from some space vector-function, and go out from standart rectangular grids to new ones (like Z – shape of virtual cells, VCM) with optimum values of parameters for sequence of superposed grids, taking into account the special properties and conditions of exactness improvement for the solution of a specific problem as in figures 2. In particular, application of a sequence of superposed grids to space-time cells differs from the original approach. So we obtain the general basic solver of transverse problem of propagation.

Last remarks

Let's mark else, that some number of sets of cells would can be received by drawing diagonal bands so, that their intersections with primaries cells form guadrates, which are inscribed or circumscribed guadrates add to primary cells. But it corresponds with a usual procedure of a refinement or an integration of primary cells, which used in methods of increase of an exactitude of solutions on a sequence of self-similar grids without rotation of them cells [1-6]. In these methods, however it is necessary to create new nodes (for refinement it's slave cells) or the part of an information is lost (for integration it's master cells). But additional prize with GSPM and VCM is quite obvious also here from using of a sequence of the grids superposition.

In addition we shall inform concerning statement of boundary conditions. In our solutions, represented in [7, 8] the same conditions were used, as with a primary cell for a new set of diagonal cells [9] and Z-shape of cells.

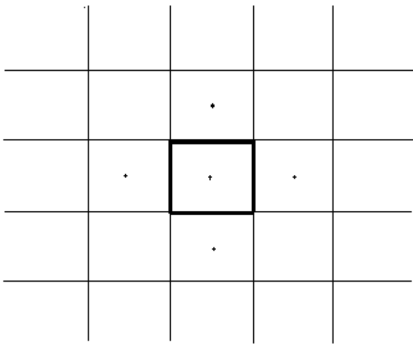

Fig. 1a. Fragment of a regular grid forming the elementary cells (guadrates). The points designate primary grid nodes of the five-dot template (2D)

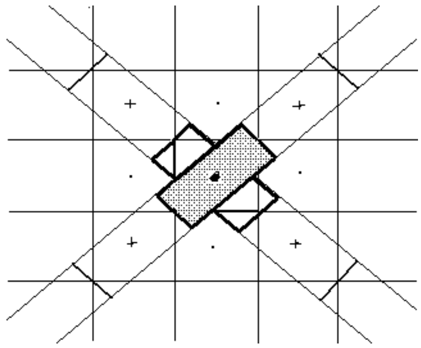

Fig. 1b. The same fragment grid, as on fig.1a; in addition diagonal bands are drawing, two from which are shown. Along each diagonal bands there are constructed the number of cells of the rectangular form with centres combined with the same primary nodes of a grids superposition.

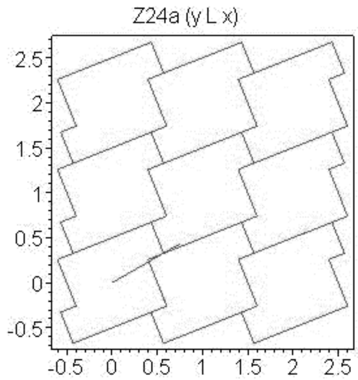

Fig. 2a.

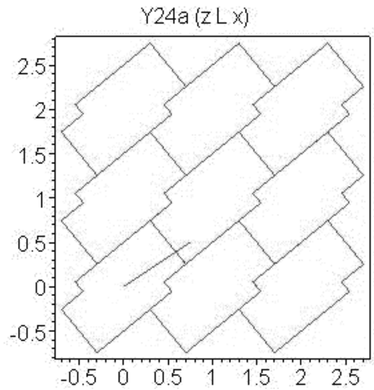

Fig. 2b.

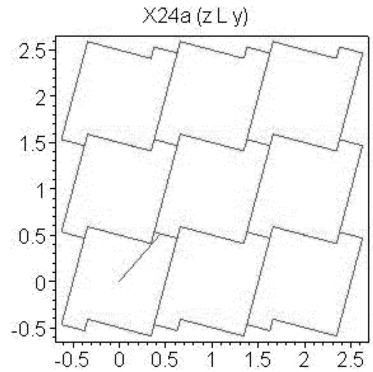

Fig. 2c.

References

1. Anderson, D.A., Tannehil, J.C. and Pletcher, R.H. 1984, Computational Fluid Mechanics and Heat Transfer, Hemisphere, New Jork.
2. Fletcher, C.A.J. 1988, Computational techniques for Fluid Dynamics, Springer–Verlag, NY.
3. LeVeque R.J. Wave propagation algorithms for multi-dimensional hyperbolic systems. J. Comput. Phys. 1997. Vol.131, pp. 327 - 353. (see also ftp://amath.washington.edu/pub/~rjl/papers/wpalg.ps.Z)
4. Marchuk, G.I. 1988, Methods of decomposition. Science, Moscou (in russian).
5. Marchuk, G.I., Schaidurov V.V. 1979, Increase of an exactitude of solutions of the difference schemes. Science, Moscow (in russian).
6. Thompson, J.F., Soni, B., Weatherill, N., 1999. Handbook of Grid Generation. CRC Press, NY.
7. Achasov O.V., Kondrashov V.V., Penyazkov O.G. Direct initiation of gaseous detonation by interacting supersonic jets. Proc. UITAM Symp. Comb. In super-sonic flows, 2-6 Oct.1995. Poitiers, France, pp. 359 - 363
8. Kondrachov V.V. About use of the simplified model of a chemical kinetics near to limits of a detonation. Ingenerno-physicheckii J., 1997. Vol.70, no.3. pp. 428 - 435.
9. Kondrashov V.V. TVD modification of a large particles method with algorithm on the superposition of grids. Ingenerno-physicheckii J., 2002. Vol.75, no. 1, pp. 41 - 48.