Parallel Algorithm for Numerical Simulation of 3D Incompressible Flows

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Numerical simulation of three-dimensional unsteady flows is a promising area of modern computational fluid dynamics. Multidimensional simulations of viscous incompressible flows became possible owing to progress in numerical methods for computing such flows in terms of the so-called natural (velocity-pressure) variables and to the high performance of multiprocessor computers [1].

The algorithm for numerical simulation of unsteady viscous incompressible flow in 3D rectangular computational domain is described. The way of parallel implementations of the algorithm is shown. The possibilities of this approach is investigated for the problem of the numerical simulation of the 3D stationary flow in cubic cavity with the upper moving wall. The problem of fluid motion in a cavity is well known and demanding benchmark test used to validate numerical techniques for flow simulation and evaluate their efficiency.

In this study the flow is simulated by using the system of quasi-hydrodynamic (QHD) equations in the version to describe viscous incompressible flows [2-3]. The explicit in time finite-difference scheme with the second order space approximation is used. Poisson equation for the pressure is each time step is solved by the iterative algorithm.

The numerical algorithm is implemented on a cluster distributed-memory computer MVS-1000M [1]. Data exchange is based on the MPI standart. The three-dimensional computational domain is partitioned into $p=p_1xp_2$ subdomains in the directions *OX* and *OY* as illustrated by Fig.1. for $p_1 = p_2 = 3$. Each processor executes computations in the corresponding geometric domain.

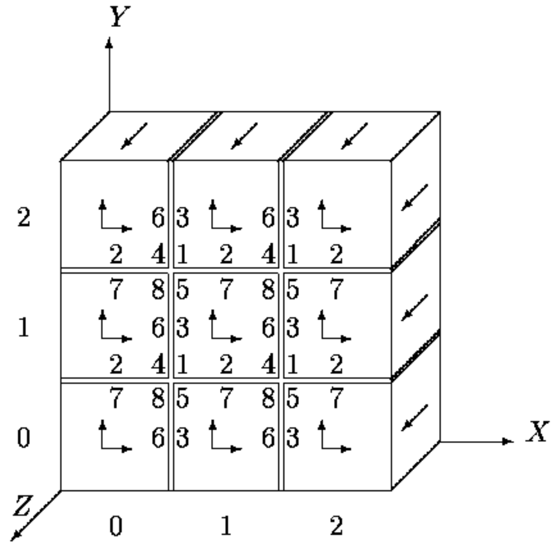

Fig. 1. Partitioning of the computational domain for the computer network.

The algorithm of parallel implementation of the solution of the explicit finite-difference scheme is well known. But the efficiency of the numerical solution is largely determined by the

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efficiency of computation of Poisson's equation for pressure. A parallel version of a modified incomplete Cholesky conjugate gradient method (MICCG(0)) is proposed for solving the second boundary value problem for the pressure [4]. Parallel version of the algorithm is based on the strategy of regulating (re-ordering) of the nodes of the computational grid. In particular, the regulating named "Domain Decomposition ordering" is used. It is proved theoretically and shown in computations that in the mentioned variant of MICCG(0) for Diriclet problem the asymptotic form of the dependence of iteration number from the number of computational points for the constant processor number remains constant. The slow increasing of the iteration steps is seen with the increasing of the processor numbers [5]. The investigations of the speed up of computations with the increasing of the processor number are made.

The flow in a cubic cavity with a moving lid is computed for the Reynolds numbers *Re*= =100, 1000 and 2000. The velocity of the lid is taken equal to u_x=1. The uniform spatial grids with equal number of grid points along all coordinates 81x81x81 and 161x161x161 are used. For *Re*=1000 the convergence of the numerical solution due to grid refinement is demonstrated.

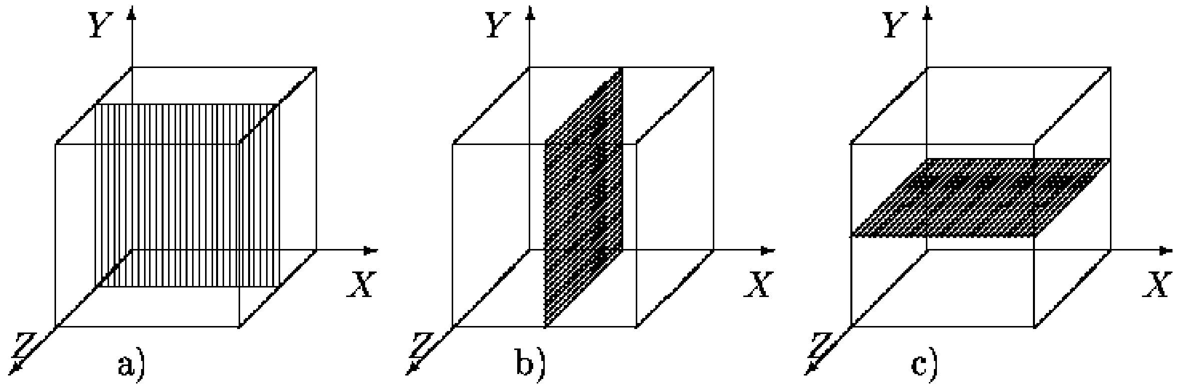

Fig. 2. Three central cross sections of the cavity.

The scheme of three central cross sections of the cavity is shown in Fig. 2. The figures 3-5 show the streamlines computed in the three central cross sections for *Re*=2000 on the grid 81x81x81. The last two of them demonstrate the substantially three-dimensional character of the flow, having the sources and sinks appear in these planes. The figures 6-8 show the one-dimensional distributions of the velocity components along *z* for *Re*=1000 and 2000. Additional details can be found in [6].

The algorithm developed in this study can be used to perform computations of three-dimensional unsteady flows in rectangular domains with reasonable time complexity.

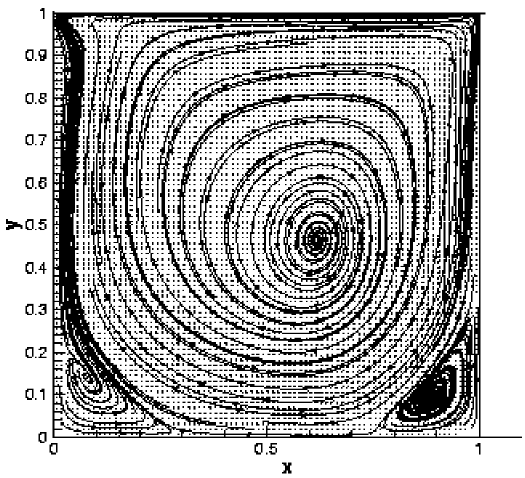

Fig. 3. *Re*=2000, streamlines for u_x, u_y in z=0.5 plane.

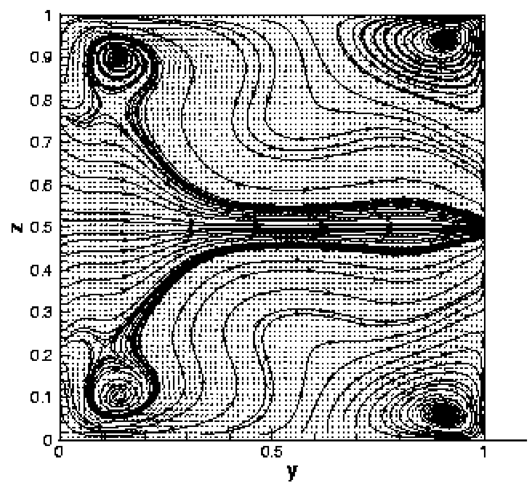

Fig. 4. *Re*=2000, streamlines for u_y, u_z in x=0.5 plane.

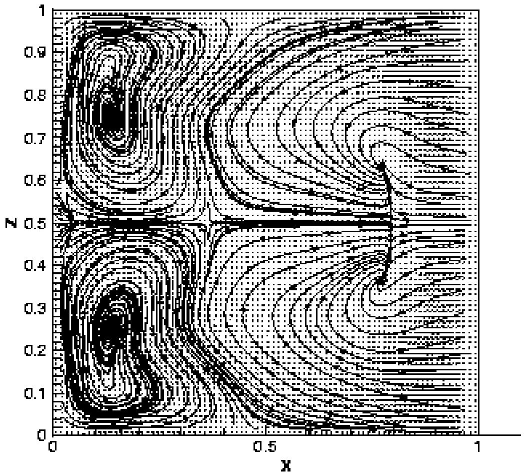

Fig. 5. *Re*=2000, streamlines for u_x, u_z in *y*=0.5 plane.

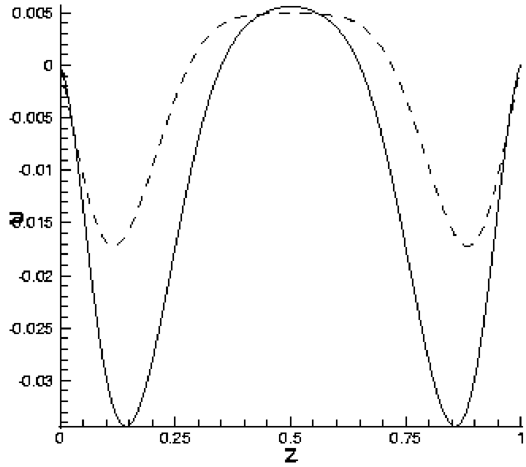

Fig. 6. u_x(0.5,0.5,z), solid line *Re*=1000, dashed line *Re*=2000.

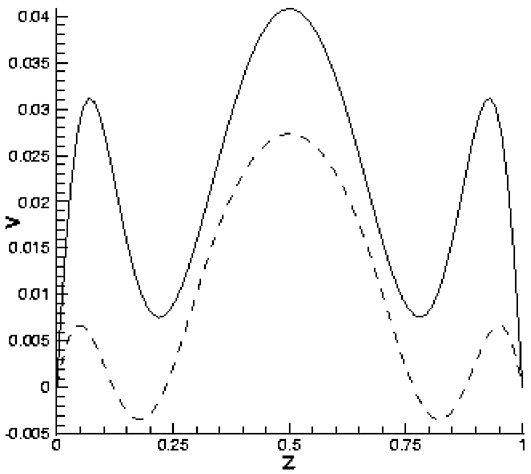

Fig. 7. u_y(0.5,0.5,z), solid line *Re*=1000, dashed line *Re*=2000.

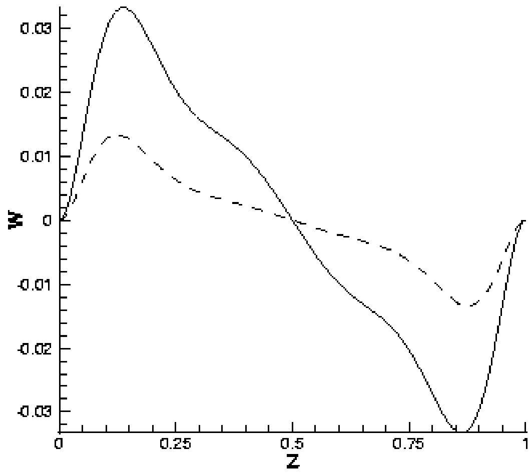

Fig. 8. u_z(0.5,0.5,z), solid line *Re*=1000, dashed line *Re*=2000.

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