Domain Decomposition Techniques to Solve the Navier-Stokes Equations

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Numerical algorithms to solve approximately problems of mathematical physics on modern parallel computers are based, as a rule, on domain decomposition techniques. In the present work there are developed domain decomposition schemes to solve numerically initial/boundary value problems for the Navier-Stokes equations in the primitive variables pressure-velocity.

In computational fluid dynamics [1] for solving 3D problems there are employed numerical algorithms based on using primitive variables pressure-velocity. The main difficulties in this approach are connected with calculation of the pressure. In studying transient problems the corresponding elliptic Neumann problem for the pressure is derived as the result of employment of one or another scheme with splitting with respect to physical processes [2].

To construct numerical algorithms for solving hydrodynamic problems on modern parallel computers, various approaches are employed [3]. In simulation of unsteady incompressible fluid flows a transition to a new time level [4] includes the solution of non-selfadjoint elliptic problems for the velocity components and evaluation of the pressure via a selfadjoint problem. Thus, parallel algorithms of computational fluid dynamics cab be constructed on the basis of parallel algorithms for solving grid elliptic problems, i.e. for solving linear algebra problems [5].

Nowadays the parallel algorithms for problems of mathematical physics are constructed on the basis of domain decomposition techniques [6] with and without overlapping of subdomains. The most advanced are studies on iterative methods of domain decomposition for selfadjoint elliptic equations of second order. For transient problems it seems to be more suitable to use non-iterative variants of domain decomposition techniques [7] which in a more full maesure employ peculiarities of a problem (evolution in time). In these region-additive schemes a transition to a new time level is performed via solving problems in particular subdomains.

In the present work issues of constructing region-additive schemes for the Navier-Stokes equations in the primitive variables are discussed. On the basis of the general theory of operator-splitting schemes [2,7] there are investigated various classes of domain decomposition schemes for problems of convection-diffusion. In simulation of incompressible flows an elliptic problem for the pressure can be replaced on separate elliptic problems for the pressure in particular subdomains. Therefore, it is possible to construct non-iterative region-additive schemes for the Navier-Stokes equations.

References

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