Aitken Acceleration of Dirichlet – Neumann Algorithm for Flow in Porous Media with Discontinuous Permeability

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The problem

In this communication, we consider the non stationary flow problem in a multi-layered aquifer. Such a geological configuration, presenting some superposed aquifers separated by layers with low permeability (figure 1), is often the physical domain in which the flow problem has to be solved [1], [2].

A second configuration which is important for some applications (as seismic research or oil recovery [3]) is also considered : it is obtained by introducing one or several faults traversing all the aquifers and supposing a landslide along the faults (figure 2).

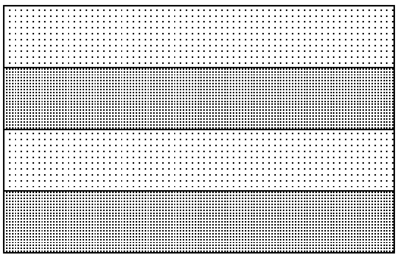

Fig. 1.

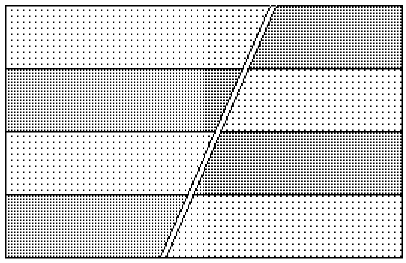

Fig. 2.

In both configurations, the permeability is a piecewise constant function, each layer being an homogeneous porous medium but with different permeability.

The groundwater flow phenomenon is governed by a parabolic equation for the fluid pressure derived from linear Darcy's equation and mass conservation law [4].

The Method

In realistic situations as the previous configurations, a natural non overlapping domain decomposition can be obtained taking into account the geological layers and using the Dirichlet-Neuman algorithm of Schwarz type. However, the distribution of the interface conditions (pressure and flow continuity) for the adjacent sub-problems is delicate since the algorithm can be divergent if the Neumann condition is not imposed for the appropriate sub-domain.

This feature can be easily shown in an one-dimensional framework, in the case of two sub-domains: the convergence of the Dirichlet-Neumann algorithm depends of the interface position and the ratio of two permeabilities. Then, since these parameters are a priori fixed, the inter-changing of the interface conditions remains the only one possibility to reach the convergence.

However, this technique is very difficult to handle in practice. Moreover, there are some situations when, in order to assure the convergence, it leads to ill-posed problem. For instance, in figure 3, we have considered three media, bounded by impermeable media in top and in bottom, and with Dirichlet conditions on lateral faces. The permeabilities of the left and the right media

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are supposed equals. Clearly, for some values of the permeability of the middle medium, to assure the convergence, one is obliged to consider only Neumann conditions on all the boundary of the middle sub-domain.

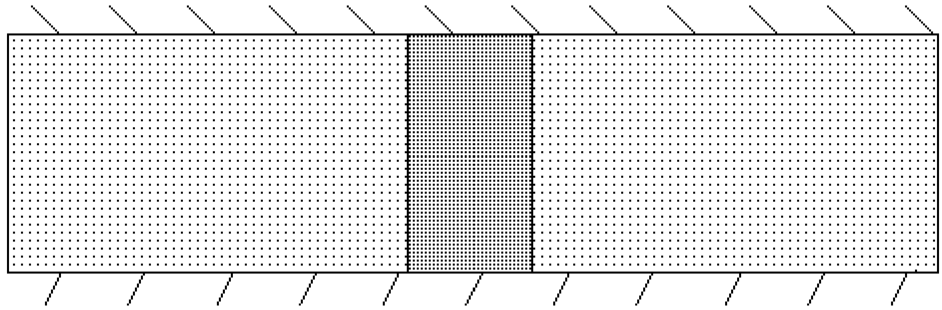

Fig. 3.

The method studied here is based on the Aitken acceleration of the iterative solutions obtained by Dirichlet-Neuman algorithm and restricted to the interfaces.

The idea of Aitken acceleration has been introduced in [5] for the classical additive Schwarz method which is linearly convergent but with low convergence rate. From numerical point of view, the authors use finite differences on regular grids and therefore, the Fourier transform is a natural tool to improve the efficiency of this algorithm.

We investigate how one can use the Aitken acceleration for the Dirichlet-Neuman algorithm which has also a linear behavior (no matter if it is convergent or divergent). We place ourselves this study in the framework of finite elements method. At continuous level, the analysis can be easily adapted. However, at discrete level, a special attention must be given to the approximation of Neumann condition (and specially to influence of the order of this approximation) and the application of Fourier transform.

For above realistic configurations some numerical experiments are presented and discussed from the parallelism point of view.

As proved in [5] for additive Schwarz method, one can expect that the Aitken acceleration of Dirichlet-Neumann algorithm is also efficient for multi-clusters computers, i.e. for two-level parallel architecture machines.

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