Parallel Algorithms of Direct Solving the Boltzmann Equation in Aerodynamics Problems

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The main idea of the direct approach that is developed by the authors is the use of schemes of the kinetic equation which are applied directly for aerodynamics complex flows with zones of different rarefaction (see [1-3]). Such direct methods for the Boltzmann equation have some advantages in comparison with combining the macroscopic and kinetic equations. The aim is to construct numerical schemes adopted for different regimes from a free-molecular one to continuum. The calculation of rarefied gas flows on the basis of the nonlinear Boltzmann equation is an extremely labour-consuming problem from the computing point of view. In this case application of multiprocessor computers is not only convenient, but also necessary tool of calculations. The use of multiprocessor computer facilities allows us to expand the class of the problems, to carry out comparison of the obtained results on a sequence of condensed grids. Parallel schemes are applied in a natural way for the direct approaches due to uniformity of numerical schemes (as a rule, the number of velocity nodes does not depend on a point in physical space).

In the general direct approach with discrete velocity scheme different procedures to evaluate collision integrals were applied: deterministic (regular) methods, Monte Carlo procedure, quasi-Monte Carlo procedure with Korobov's sequences of nodes. To guarantee the conservativity of schemes two approaches were used. In the first approach the conservative splitting method was applied to produce the so-called hydrodynamic conservation of the numerical scheme for the Boltzmann equation. The second approach (the kinetic conservativity) is the conservative discrete ordinate method providing the conservation laws for the full collision integral (an analogous method was used in [4]).

Monte Carlo and quasi-Monte Carlo methods for evaluation collision integrals are ordinary procedures now and described in [1]. In one of the deterministic approaches the exact integration over impact angles is carried out. Molecular models are studied for which a semi-analytical method is directly applicable (e.g., a hard sphere model, VHS model). The formal scheme of the finite element method in velocity space is used. The important feature (small number of parameters) makes the algorithm quite simple for multi-dimensional problems. In fact, the problem of calculation is related to computation of a huge number of elements of matrices of the quadratic form.

Also a new deterministic "node-to-node" method of collision accounting was used (parallel schemes are desirable in this expensive algorithm of an evaluation of the Boltzmann integral without interpolation in velocity space). In this method a spherical computational domain in velocity space is considered, and only collisions with the collision sphere belonging to computational domain are taken into account. Given the distribution function in the obtained nodes the collision frequency and the integral of inverse collisions is computed. For comparison of results the method of discrete ordinates [4] was also considered.

In accordance with the method of solving the kinetic equation appropriate parallel algorithms were applied. Algorithms can be considered as two main stages – a free-molecular flow and relaxation. For Monte Carlo, quasi-Monte Carlo approaches and for the "node-to-node"

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deterministic method an effective way is to use a paralleling in physical space. For the uniform relaxation stage, which is the resource consuming part, an evaluation of collision integrals for an each point needs the same number of operations. Thus this stage is ideally parallelized. For a free-molecular flow stage it needs to exchange information between neighbor points in physical space but the computing and exchange time of a free-molecular stage is negligible in comparison with the uniform relaxation stage. General features of the algorithm are ideal load balancing, low data-exchange time (compared to computing time) and negligible number of duplicated operations. High efficiency of algorithms was confirmed by numerical experiments.

For the deterministic method with integration over impact parameters a parallelization in velocity space is used because it is necessary to distribute a huge quadratic form matrix between processors. The characteristic features of this method lead to good efficiency of parallelization in velocity space in contrast with the Monte Carlo evaluation approaches. The realization of the algorithm for the relaxation problem demonstrated good parallel performance. This allows us to conclude that speed-up will be at least the same for spatial problems (because free-molecular flow stage is ideally parallized in this case).

The main problems under consideration were: 2D and 3D supersonic free jet flows; a 2D normal impact of a supersonic jet. A class of supersonic jets is important from both theoretical and practical points of view. Such problems are relatively simple for the direct solving the Boltzmann equation because of supersonic character of a flow, not very high Mach numbers, simple boundary conditions for free flow surfaces etc.

Free jet flow problems are studied by several methods under consideration. A 2D supersonic underexpanded free jet issuing from a slit, and 3D jet issuing from a circular orifice are studied with the aspect (pressure) ratio equal to about 10. In this case the jet structure is mainly determined by a single barrel structure with a single Mach disc, and a computational domain was not very long. A sonic nozzle was considered. Jet flows were investigated in a wide range of rarefaction from a free-molecular flow up to small Knudsen numbers Kn=0.001.

A special attention was paid to study unstable turbulent pulsation in mixing layer in a free jet at supercritical regimes obtained in computations in [1] and compared with the experimental data [6]. These calculations are now are tested in detailed considerations with kinetic time scales. The rarefaction factor was considered in a supercritical range: Kn=0.00003-0.0001.

An efficiency of the present methods [2] is illustrated in solving a 2D problem of impact of the plane steady jet on a solid wall. The formulation of the problem is the same as in [5]. A plane x-directed jet is considered. The sound nozzle of height H is arranged at $x=0$. The gas jet enters into a space containing a gas at rest, with temperature T equal at initial moment to the stagnation temperature of a jet. The pressure of a gas at initial moment is 0.015 of the stagnation pressure. For a case represented in report the average free length on an edge of a nozzle is equal $0.0012H$. The diffuse reflection from a wall is used, and a mono-atomic hard sphere gas is considered.

Calculations were performed on parallel multiprocessor computers PARSYTEC CC (8 processors) in Institute for Mathematical Modelling of Russian Academy of Sciences and on HP and MVS1000 (16, 32 and 64 processors) in Joint Supercomputer Center of Russia.

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