Lattice Boltzmann Model for Compressible Flows

Andrew T. Hsu[a*], Tianliang Yang[a], Sule Celasun[a], Isaac Lopez[b], and Akin Ecer[a]

[a]Department of Mechanical Engineering,
Indiana University - Purdue University, Indianapolis
Indianapolis, IN 46202-5132, USA

[b]NASA Glenn Research Center, Cleveland, Ohio 44135, USA

Key words: lattice Boltzmann, compressible flow, three-dimension, Euler equation, Navier-Stokes equation.

This paper gives a review of the lattice Boltzmann method for compressible flows. The lattice Boltzmann method (LBM) [1-3] is a relatively new numerical approach for simulating complex flow and transport phenomena in cases where direct solution of the Navier-Stokes equations is not practical. Unlike conventional CFD method based on macroscopic continuum equations, the LBM uses a mesoscopic equation, i.e., the Boltzmann equation, to determine macroscopic fluid dynamics. The LBM is flexible, has broad applicability, and may be easily adapted for parallel computing. It has been successfully applied to multiphase and multi-component fluids, flows through porous media, and solid particle suspensions.

The LBM originated from a Boolean model known as the lattice gas automata (LGA) [4-5]. In a LGA method, the local equilibrium distribution is described by the Fermi-Dirac statistics. As a result, LGA has several shortcomings: high statistical noises, the violation of Galilean transformation invariance in their resulting hydrodynamics equations, and the failure in high Reynolds number computations. To eliminate noise, the Boltzmann equation was used to simulate the lattice gas automata [6-7], however, other problems, i.e. non-Galilean invariance and low Reynolds number, remained. These difficulties led to the development of the LB method [1-3]. Higuera et al. and Benzi et al. [3] simplified the collision term by a linear operator. Chen et al. [1] and Qian et al. [2] used a simpler collision operator of the BGK type [8]. The equilibrium distribution was an approximation of the Maxwellian equilibrium distribution. Galilean invariance was guaranteed in these LB models. The LB models of BGK type [1,2] have only a single ratio of viscosity to thermal conductivity, while the models of linear collision operator [3] allow for independently varying viscosities and thermal conductivities. The LB models have been successfully applied to various physical problems, such as single component hydrodynamics, multiphase and multi-component fluid flows, magneto-hydrodynamics, reaction-diffusion systems, flows through porous media, and other complex systems at small Mach numbers [9-10].

Unfortunately, as a new CFD tool, the general LB method developed in the past suffered from the constraint of small Mach number because the particle velocities belong to a finite set, and the resulting macroscopic velocity is always much smaller than the speed of sound calculated from the microscopic diffusion velocity.

Efforts have been made to increase the allowable Mach number range and to incorporate the effects of temperature into lattice Boltzmann simulations. Choosing a modified equilibrium distribution, Alexander et al. [11] replicated the Burger's equation with a controllable sound speed. Yu and Zhao [12] introduced an attractive force to reduce the sound speed and to alleviate the small Mach number restriction; however, the energy equation was not recovered in their formulation. Palmer and Rector [13] formulated a thermal model that can simulate temperature variations in a flow, but high Mach number effects were not included in that study.

*E-mail: anhsu@iupui.edu

Two schemes related to the LB method, the gas-kinetic theory [14,15] and the discrete-velocity model [16,17], had been used to simulate the compressible Euler equations. In both of the above work, the flux-splitting approach with TVD flux limitation was employed to determine the mean flux to neighboring cells. Nadiga [16,17] introduced an adaptive-velocity concept in the discrete-velocity model for compressible inviscid flows. Huang et al. [18] similarly use adaptive discrete velocities to simulate one-dimensional shock waves. Only under special circumstances, the Boltzmann equation used in these methods is equivalent to the lattice Boltzmann equation [19], but the lattice Boltzmann equation is much easier to solve.

Recently, we proposed a locally adaptive LB model on hexagonal lattice [20] based on a large particle-velocity set so that mean flow may have a high velocity; however, the support set of the equilibrium distribution is quite small and similar to the adaptive velocities of Nadiga's Euler solver [17]. This model is suitable for a wide range of Mach numbers and does not consume much computer resource. Compressible Navier-Stokes equations including the energy equation are derived from the BGK lattice Boltzmann equation; therefore, this model can simulate compressible viscous flows that include heat transfer [21,22]. If the viscous terms are considered as discretization error and a slip wall condition is employed, the solution can be compared with compressible Euler solutions. The numerical simulations showed that the model has the capability of solving compressible Euler flows with strong shocks [20,23,24] and has high parallel efficiency [25,26]. This locally adaptive LB model has been also formulated on a two-dimensional square lattice [27]. All the previous simulations were carried out for periodical boundary or flat wall boundary, or a combination of the two.

In LBM, the boundary conditions have been directly adopted from the lattice gas automaton method. A common method of modeling no-slip walls in LBM simulations is to use the bounce-back boundary condition in which particles that stream into the walls "bounce back" and exit the wall in the direction from which they came. It has been noted that the bounce-back boundary condition is 2nd order for walls aligned with the lattice, however, it gives only first order accuracy at the curved boundaries [28,29]. Several boundary treatments have been proposed for achieving second order accuracy for no-slip velocity conditions on curved walls [30-33]. In these treatments boundary conditions for the particle distribution function had to be handled with given macroscopic quantities. In complicated fluid flows, boundary conditions might include a combination of velocity, density, temperature, and their derivatives. To a certain degree, achieving self-consistent boundary conditions with a given accuracy is as important as developing numerical schemes themselves.

We have recently proposed a three-dimensional compressible LB model on a square lattice. A large particle-velocity set is used to enable the simulation of high Mach number flows. Meanwhile, in order to make the computation more tractable, a small support set for the equilibrium distribution is employed. This model can handle flows over a wide range of Mach numbers and capture jumps through shock waves. Due to the simple form of the equilibrium distribution, the 4th-order velocity tensors are not involved in the calculations. Unlike the standard lattice Boltzmann model on square lattice, there is no need of special treatment for the homogeneity of 4th-order velocity tensors. Therefore, the Navier-Stokes equation and energy equation were recovered with only 6 symmetric particle velocity directions. The second-order discretization errors in velocity have been eliminated to improve the accuracy in viscous flows simulations. The model is valid for both viscous and inviscid compressible flows with or without shocks.

The present scheme deals only with the equilibrium distribution that depends on fluid density, velocity, and internal energy only. We proposed a boundary condition based on an extrapolation of the macroscopic variables for curved walls. This boundary condition treatment is self-consistent, easy to implement, and suitable for both slip wall and non-slip wall boundary conditions. Moreover, it can be easily extended to complex flows with moving walls, mass injection from the walls, and heat exchange with the walls.

References to the abstract will be provided in the full paper.